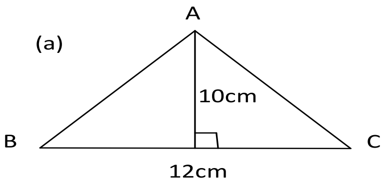
# MENSURATION (AREA, SURFACE AREA AND VOLUME)

## AREA OF TRIANGLES

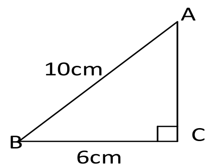
### (i) Given the base and perpendicular height

1. Find the area of the triangles below

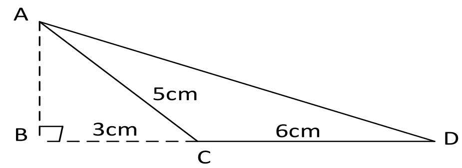
(a)




(b)




(c)




**Solution**

(a) $A = \frac{1}{2}bh = \frac{1}{2}x12x10 = 60cm^2$

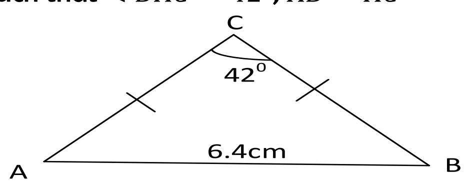
(b) $h = \sqrt{10^2 - 6^2} = \sqrt{64} = 8cm$

$$A = \frac{1}{2}bh = \frac{1}{2}x6x8 = 24cm^2$$

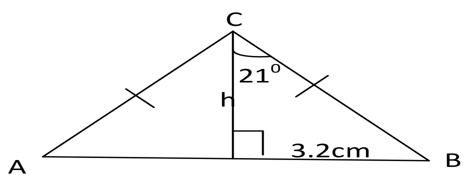
(c) $h = \sqrt{5^2 - 3^2} = \sqrt{16} = 4cm$

$$A = \frac{1}{2}bh = \frac{1}{2}x6x4 = 12cm^2$$

2. Find the area of a triangle ABC such that $< BAC = 42°$, $AB = AC$




**Solution**

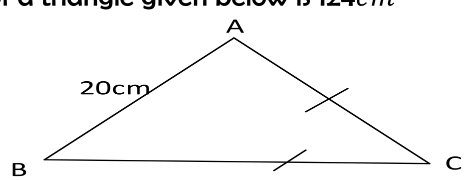



$$tan21 = \frac{3.2}{h}$$

$$h = \frac{3.2}{tan21°} = 8.34cm$$

$$A = \frac{1}{2}bh = \frac{1}{2}x6.4x8.34 = 26.69cm^2$$

3. The area of a triangle given below is 124$cm^2$

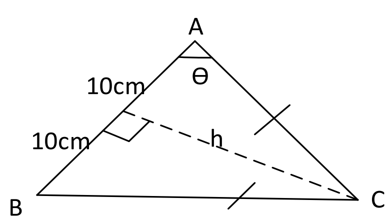



Find;

(a) Angle BAC

(b) Length AC

**Solution**




$$A = \frac{1}{2}bh$$

$$\frac{1}{2}x20xh = 124$$

$h = 12.4cm$

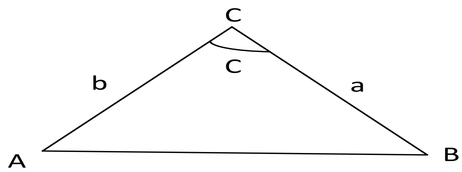
$$tan\theta = \frac{12.4}{10}$$

$$\theta = tan^{-1}\left(\frac{12.4}{10}\right) = 51.12°$$

$$cos51.12° = \frac{10}{AC}$$

$$AC = \frac{10}{Cos\ 51.12°} = 15.93cm$$

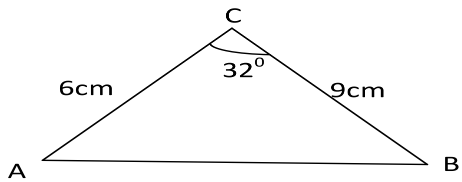
### (ii) Given two sides and the angle between them




$$\boxed{A = \frac{1}{2}ab\ sinC}$$

1. Find the area of a triangle ABC such that $AC = 6cm$, $BC = 9cm$ and $< BCA = 32°$

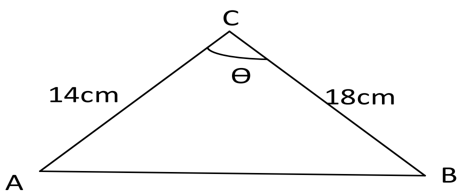
**Solution**




$$A = \frac{1}{2}ab\ sinC$$
$$A = \frac{1}{2}x9x6\ sin32° = 14.31cm^2$$

2. The area of a triangle is 72.4cm, if the two of its sides are 14cm and 18cm, find the included angle

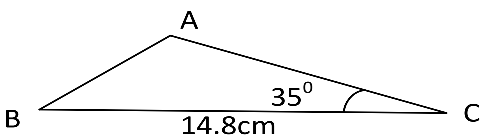
**Solution**




$$A = \frac{1}{2}absinC$$

$$72.4 = \frac{1}{2}x14x18sin\theta$$
$$sin\theta = \frac{72.4x2}{14x18}$$
$$\theta = sin^{-1}\left(\frac{72.4x2}{14x18}\right) = 35.07°$$

3. The area of the triangle PQR is $24.41cm^2$

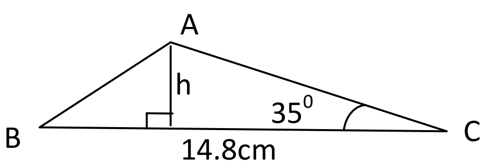



Find;

(a) The length AC

(b) Shortest distance between A and line BC

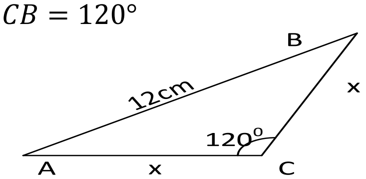
**Solution**




$$A = \frac{1}{2}absinC$$

$$24.41 = \frac{1}{2}x14.8xACsin35°$$
$$AC = 5.75$$
$$A = \frac{1}{2}bh$$
$$\frac{1}{2}x14.8xh = 24.41$$
$$h = 3.299cm$$

4. The figure below shows an isosceles triangle ABC in which $AB = 12cm$, $AC = BC = xcm$ and angle $ACB = 120°$




**Find;**

(a) Value of x

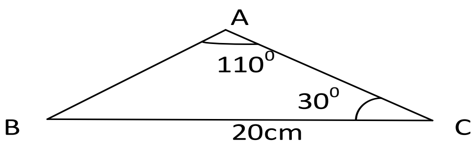
(b) Area of the triangle

**Solution**

$$12^2 = x^2 + x^2 - 2x^2cos120°$$
$$144 = 2x^2(1 - cos120°)$$
$$2x^2 = \frac{144}{1.5}$$
$$2x^2 = 96$$

$$x^2 = 48$$
$$x = \sqrt{48} = 6.93cm$$
$$A = \frac{1}{2}x6.93^2sin120° = 20.8cm^2$$

5. The figure below shows a triangle ABC in which $BC = 20cm$, and angle $ACB = 30°$

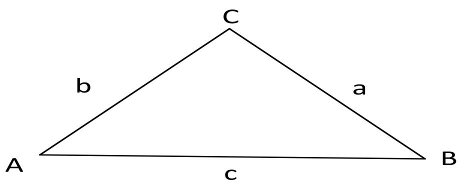



**Find;**
(a) Length of AB
(b) Area of the triangle

**Solution**

$$\frac{AB}{sin30}=\frac{20}{sin110}$$
$$AB=\frac{20}{0.9397}x0.5$$
$$AB=10.64cm$$
$$A=\frac{1}{2}x10.64x20xsin40°=68.1cm^2$$

(iii) **Given all the three sides**

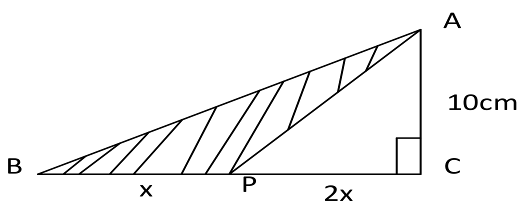



$$s=\frac{a+b+c}{2}$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

1. A triangle has sides of length 10cm, 14cm and 17.2cm. find its area
**Solution**
$$s=\frac{a+b+c}{2}=\frac{10+14+17.2}{2}=20.6cm$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}$$
$$A=\sqrt{20.6(20.6-10)(20.6-14)(20.6-17.2)}$$
$$A=69.99cm^2$$
2. A maize farm in the shape of a triangle. Its dimensions are 480m by 520m by 640m. if production per hectare is approximately 40 bags of maize, find the number of bags of maize the owner expects at the end of harvesting season to the nearest bag
**Solution**
$$s=\frac{a+b+c}{2}=\frac{480+520+640}{2}=820cm$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}$$
$$A=\sqrt{820(820-520)(820-640)(820-480)}$$
$$A=22699cm^2$$
$$A=\frac{22699}{10000}ha=12.2699ha$$
$$total\ bags=12.2699x40=490.796$$
$$=491\ bags$$

**Other examples**

1. In the given triangle ABC, the shaded area is 20c$m^2$. Given that $AC=10cm$, $BP=xcm$ and $PC=2x$ cm, find the un shaded area




**Solution**

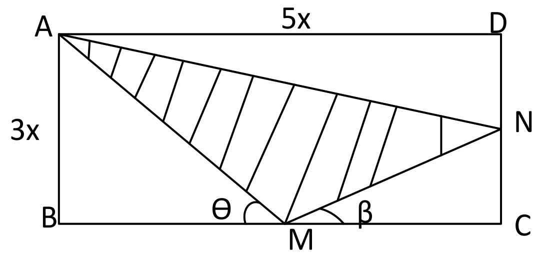
Total area= $\frac{1}{2}x3xx10=15xcm^2$

Un shaded area= $\frac{1}{2}x2xx10=10xcm^2$

Shade area= $15x-10x=5x$

$$5x=20$$
$$x=4$$
Un shaded area= $10x4=40cm^2$

2. In the figure ABCD is a rectangle in which AD= $5x\ cm$ and AB= $3x\ cm$. M and N are the mid points of BC and CD respectively.




(a) Show that the area of the shaded region is $\frac{45x^2}{8}cm^2$

(b) Given that the area of the triangle MNC= $30cm^2$, find the dimensions of the rectangle

(c) Calculate the size of angles;
   (i) $\theta$
   (ii) $\beta$
   (iii) AMN

**Solution**

**(a)** $\Delta ABM;\ A = \frac{1}{2}(3x)(2.5x) = \frac{15x^2}{4}$

$\Delta MNC;\ A = \frac{1}{2}(1.5x)(2.5x) = \frac{15x^2}{8}$

$\Delta ADN;\ A = \frac{1}{2}(5x)(1.5x) = \frac{15x^2}{4}$

Total un shaded area= $\frac{15x^2}{4} + \frac{15x^2}{8} + \frac{15x^2}{4} = \frac{75x^2}{8}$

Area of rectangle $= (5x)(3x) = 15x^2$

Shaded area= $15x^2 - \frac{75x^2}{8} = \frac{45x^2}{8}$

**(b)** $\frac{15x^2}{8} = 30$

$15x^2 = 240$

$x^2 = 16$

$x = 4cm$

$AD = BC = 5x4 = 20cm$

$AB = CD = 3x4 = 12cm$

**(c)** $tan\theta = \frac{12}{10}$

$\theta = tan^{-1}(1.2) = 50.2°$

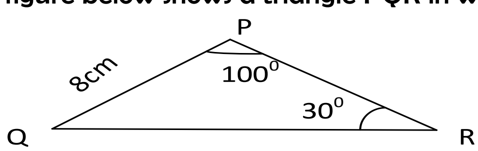
$tan\beta = \frac{6}{10}$

$\beta = tan^{-1}(0.6) = 31°$

$< AMN = 180 - (50.2 + 31) = 98.8°$

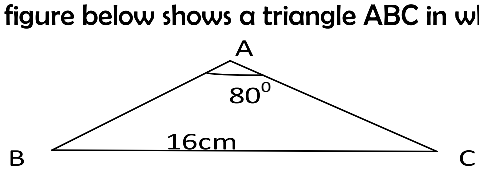
**Exercise**

1. In a triangle ABC, angle BAC= $150°$, AB= $5cm$ and AC= $4cm$. Calculate the area of the triangle ABC. **An**($5cm^2$)
2. An equilateral triangle PQR has an area of 97.43$cm^2$. calculate the length of each side of the triangle **An**(15cm)
3. Calculate the areas of a triangle PQR in which $QR = 15m$, angle $Q = 70°$ and $R = 80°$ **An**(208.2$m^2$)
4. The figure below shows a triangle PQR in which $PQ = 8cm$, $QPR = 100°$ and angle $QRP = 30°$



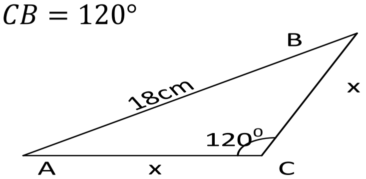

**Find;**
(a) Length of QR
(b) Area of the triangle
**An**($(a) = 13.74cm,, (ii) = 38.86cm^2$)

5. The figure below shows a triangle ABC in which $BC = 16cm$, $BAC = 80°$



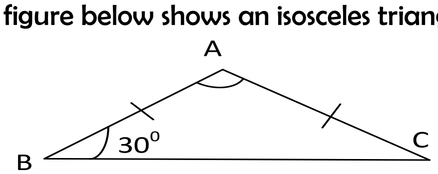

**Find;**
(a) Length of AB
(b) Area of the triangle
**An**($(a) = 12.45cm,, (ii) = 76.3cm^2$)

6. The figure below shows an isosceles triangle ABC in which $AB = 12cm$, $AC = BC = xcm$ and angle $ACB = 120°$



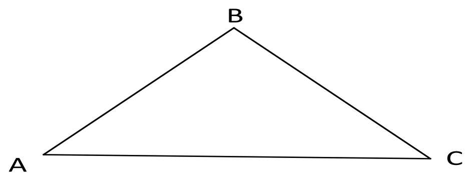

**Find;**
(a) Value of x
(b) Area of the triangle
**An**($(a) = 10.39cm,, (ii) = 46.74cm^2$)

7. The figure below shows an isosceles triangle ABC in which $AB = AC$and angle $ABC = 30°$




Given that the area of the triangle is $96.96cm^2$, calculate the length of AB
**An**($AB = 15cm,,$)

8. In the triangle below, $AB = 5cm$, $BC = 6cm$ and angle ABC is 60°.




Find;

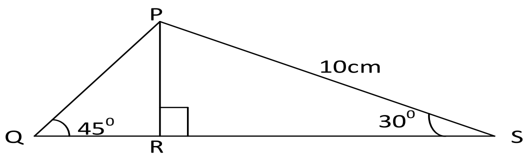
(i) AC

(ii) The remaining angles of the triangle

(iii) Area of triangle ABC

**An**($(i)$ =5.567cm, $(ii)\ ACB = 51°, < BAC = 69°, (iii) = 12.99cm^2$)

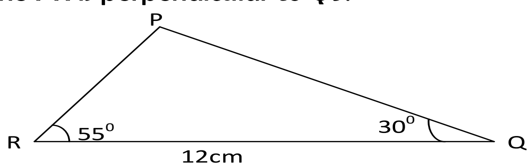
9. The figure below shows triangle PQS in which $PS = 10cm$, angle $PSQ = 30°$and angle $PQS = 45°$. Line PR is perpendicular to QS.




Calculate the area of triangle PQS

**An**(34.15$cm^2$)

10. The figure below shows triangle PQS in which $QR = 12cm$, angle $PQR = 30°$and angle $PRQ = 55°$. Line PR is perpendicular to QS.



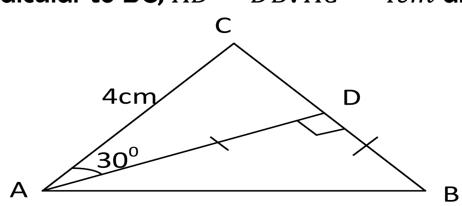

Calculate the;

(i) Length of PQ

(ii) area of triangle PQS

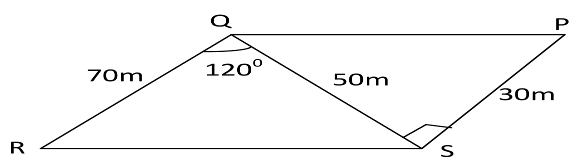
**An**($(i)$ =9.87cm, $(ii) = 29.6cm^2$)

**11.** In the figure below AD is perpendicular to BC, $AD = DB$. $AC = 4cm$ and angle $CAD$ =30°. Find AB



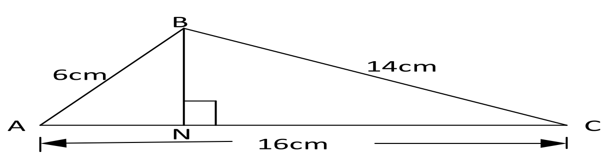

**An**(4.899cm)

12. In the figure below PQRS is a quadrilateral with angle PSQ is 90° and angle SQR= 120°. Find the area of the figure




**An**($2265.5m^2$)

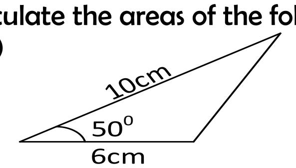
13. In the figure below BN is perpendicular to AC. Find the ratio of the area of triangle ABN to that of ABC



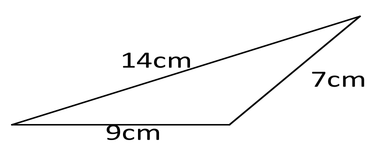

**An**($3: 13$)

14. Calculate the areas of the following triangles

(a)

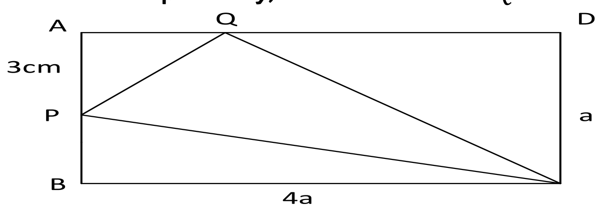



(b)




**An**($(a) = 22.98cm^2, (b) = 26.83cm^2$)

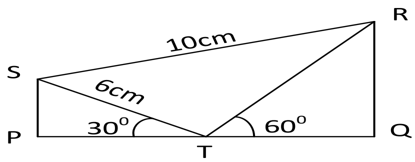
15. In the diagram below ABCD is a rectangle in which $BC = 4acm$ and $CD = acm$. P and Q are points on AB and AD respectively, such that $AP = AQ = 3cm$




(i) Find the sum of the areas of triangles BCP and CDQ in terms of a
(ii) Given that the area of triangle PQC is $40.5cm^2$, find the value of a
(iii) Express the area of triangle PCQ as a ratio of the area of the rectangle ABCD

**An**$((i) = \frac{8a^2-15a}{2}, (ii)\ a = 6cm, (iii) = 9:32)$

16. In the diagram below shows a trapezium PQRS in which PS is parallel to QR, $SR = 10cm$ and angle $PQR = 90°$. T is a point on PQ such that $ST = 6cm$, angle $PTS = 30°$ and angle $QTR = 60°$



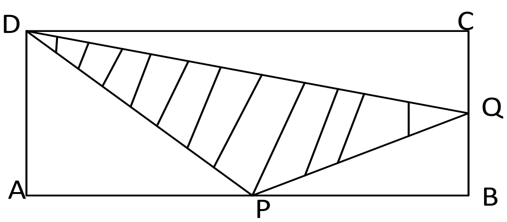

(a) Find the size of angle STR

(b) Calculate the length of;
(i) TR
(ii) QR
(iii) PS
(iv) PQ

(c) Determine the area of the trapezium PQRS

**An**$((a) = 90°\ (b)\ (i) = 8cm, (ii) = 6.9cm, (iii) = 3cm\ (iv) = 9.2cm, (c) = 45.54cm^2)$

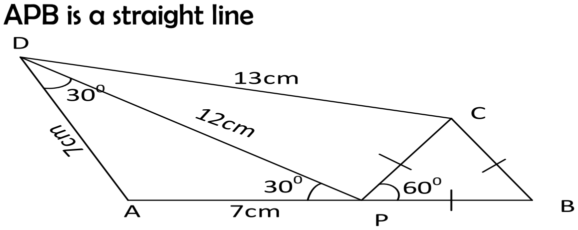
17. In the figure ABCD is a rectangle in which BC= $2x\ cm$ and AB= $3x\ cm$. P and Q are the on AB and BC respectively such that $AP = \frac{3}{4}AB$ and $BQ = \frac{2}{3}BC$




(a) Show that the area of the triangle APD is equal to the area of triangle PQD
(b) Given that the area of the shaded region is $36cm^2$, determine the value of x and state the dimensions of the rectangle
(c) Calculate the size of angle ADP;

**An**$((a)x = 4, AB = 12cm, BC = 8cm\ (b) = 48.4°)$

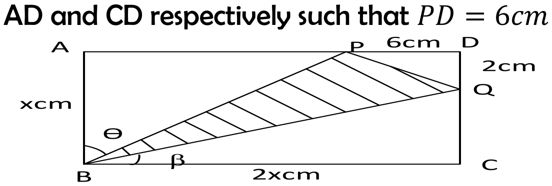
18. The diagram below shows an equilateral triangle PBBC joined to two other triangles to form quadrilateral ABCD, $PD = 12cm, CD = 13cm$ and $AP = AD = 7cm$. Angle $ADP$ =angle $APD = 30°$ and APB is a straight line




(a) Determine the size of angle DPC and hence calculate the length of PC
(b) Calculate the area of the quadrilateral

**An**$((a) = 90°, PC = 5cm(b) = 62cm^2)$

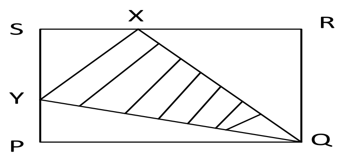
19. The figure below shows a rectangle ABCD in which $AB = xcm$ and $BC = 2xcm$. Points P and Q are on AD and CD respectively such that $PD = 6cm$ and $DQ = 2cm$




(a) Show that the area of the shaded region is $(5x - 6)cm^2$
(b) Given that the area if triangle ABP is $40cm^2$, find the value of x and hence calculate the area of the shaded region
(c) Find the size of the marked angles

**An**$((b)x = 8, A = 34cm^2, (c)\ \theta = 51.35°, \beta = 20.55°)$

20. In the figure PQRS is a square of side $lcm$ points X and Y are on SR and SP respectively such that $SX = \frac{1}{3}SR$ and $SY = \frac{2}{3}SP$.




(a) Show that the area of the shaded region is $\frac{7}{18}l^2cm^2$

(b) Show that the sum of the areas of triangles SXY and QXY is equal to half the area of the square

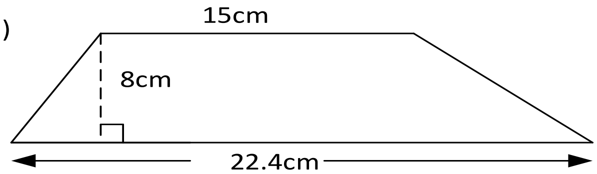
(c) Given that the area of the shaded region is 56$cm^2$, find the value of $l$ and hence the area of triangle SXY

**An**((c) l= $12cm, A = 16cm^2$)

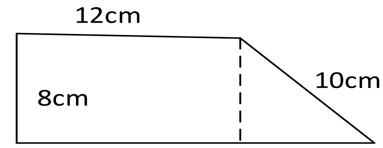
## AREA OF TRAPEZIUM

1. Find the area of the trapezium given below

(a)




(b)

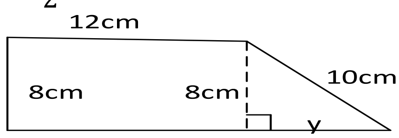



**Solution**

(a) Area of trapezium= $\frac{1}{2}(a + b)h$

$$= \frac{1}{2}(15 + 22.4)x8 = 149.6cm^2$$

(b)




$$y = \sqrt{10^2 - 8^2} = 6cm$$

Area of trapezium= $\frac{1}{2}(a + b)h$

$$= \frac{1}{2}(12 + 18)x8 = 120cm^2$$

2. The longer parallel side of a trapezium is twice as long as the shorter parallel side. The perpendicular distance between the parallel sides is 10cm. if the area of the trapezium is 225$cm^2$, find the the length of the longer parallel side

**Solution**

Area of trapezium= $\frac{1}{2}(a + b)h$

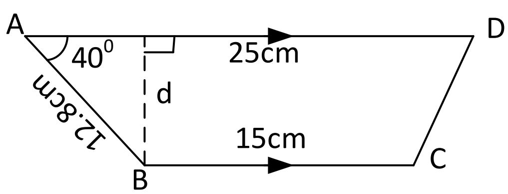
$$= \frac{1}{2}(x + 2x)x10 = 225cm^2$$

$$30x = 450$$

$$x = 15cm$$

Longer side= $2x = 2x15 = 30cm$

3. In figure, ABCD is a trapezium in which AD is parallel to BC. Given that $AD = 25cm$, $BC = 15cm$, $AB = 12.8cm$ and angle DAB= $40°$. Calculate the area of the trapezium.



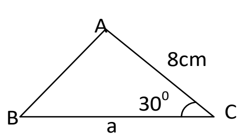

**Solution**

$$sin40 = \frac{d}{12.8}$$

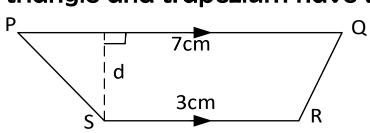
$$d = 8.228cm$$

Area of trapezium= $\frac{1}{2}(15 + 25)x8.228 = 164.554cm^2$

4. Figure (i) shows a triangle ABC in which AC= $8cm$, BC= $acm$ and angle ACB= $30°$. Figure (ii) shows a trapezium PQRS in which PQ= $7cm$, SR= $3cm$, PQ is parallel to SR and the distance between them is dcm. Given that the triangle and trapezium have the same area, determine the ratio of $d: a$




(i)




(ii)

**Solution**

Area of triangle= $\frac{1}{2}8asin30 = 2a$

Area of trapezium= $\frac{1}{2}(7 + 3)d = 5d$

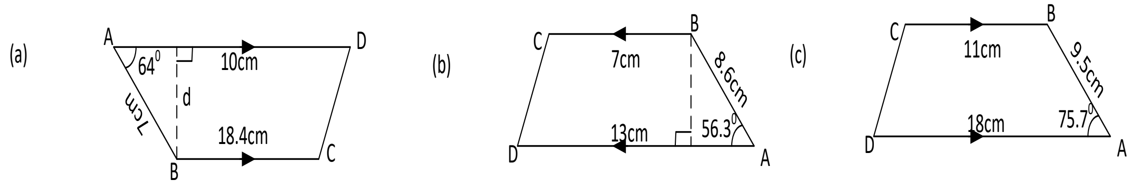
$$5d = 2a$$

$$\frac{d}{a} = \frac{2}{5}$$

$$d: a = 5: 2$$

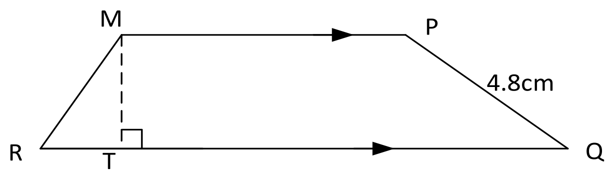
**Exercise**

1. Find the area of the figure below




**An**$((a) = 89.34cm^2\ (b) = 71.55cm^2, (c) = 133.5cm^2)$

2. MPQR is a trapezium whose area is 25$cm^2$. Given that MP$= 6cm$, PQ$= 4.8cm$ and RQ$= 8.4cm$




Find;

(a) PT

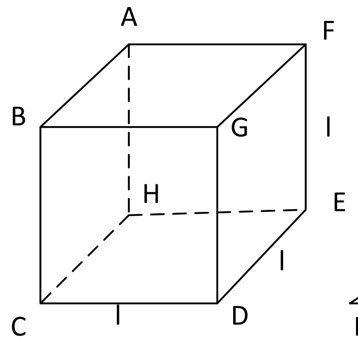
(b) Angle MPQ

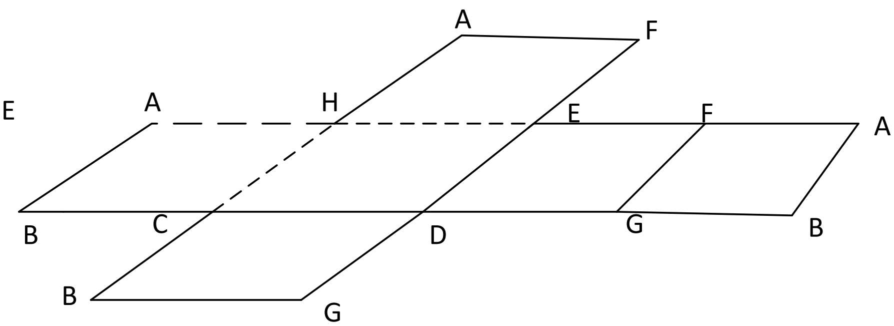
**An**$((a) = 3.33cm\ ,\ (b) = 136.02°)$

3. The longer side of a trapezium is three times as long as the shorter parallel side. The perpendicular distance between the parallel sides is 15cm. if the area of the trapezium is 180 $cm^2$calculate the length of tis longer parallel side **An**$(18cm)$

## SURFACE AREA

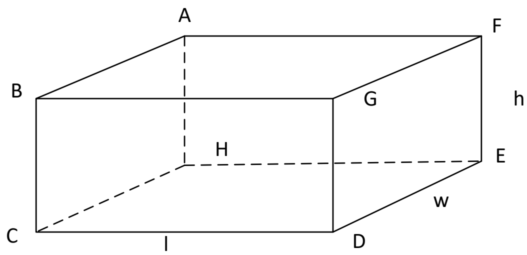
### (i) Surface area of cube

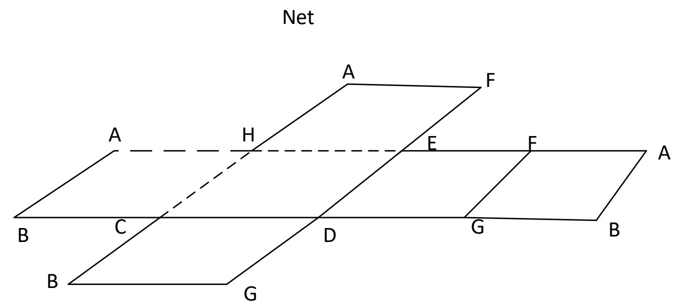



Surface area of a cube= $6l^2$

### (ii) Surface area of cuboid

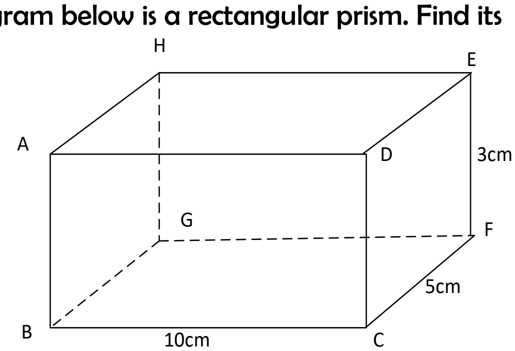



Surface area of a cuboid $= 2(lw + wh + lh)$

**Examples**

1. The diagram below is a rectangular prism. Find its surface area



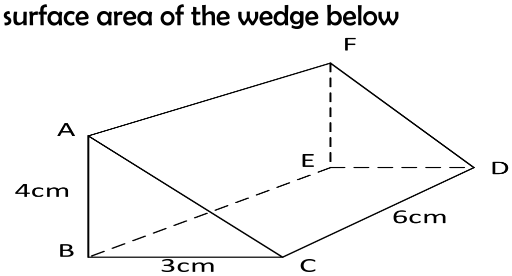

**Solution**

Surface area of a cuboid $= 2(lw + wh + lh)$

$$= 2(10x5 + 10x3 + 5x3)$$

$$= 190cm^2$$

2. Find the surface area of the wedge below



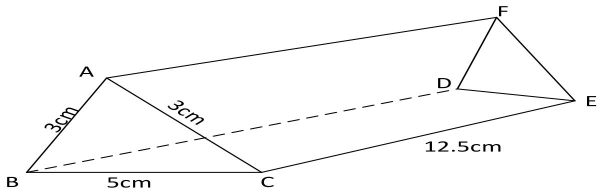

**Solution**

$$AC = \sqrt{4^2 + 3^2} = 5cm$$

$$T.S.A = 2x\frac{1}{2}x4x3 + 6x3 + 6x4 + 5x6$$

$$= 84cm^2$$

3. The diagram below shows a glass prism ABCDEF as shown below




$DC = EB = 12.5cm$, $BC = 5cm$ and $AC = AB = 3cm$

Calculate;

(a) Total surface area of the prism
(b) Size of the angle;
  (i) Between the planes ACEF and BCED
  (ii) Between ACEF and ABFD

**Solution**

(a) Let m be the midpoint of BC

$$AM = \sqrt{3^2 - 2.5^2} = 1.66cm$$

Area of $\Delta = 2x\left(\frac{1}{2}\ x1.66x5\right) = 8.3cm^2$

Area of $rectangles = 2x(3\ x12.5) = 75cm^2$

Area of $rectangle = (12.5\ x5) = 62.5cm^2$

T.S.A$= 8.3 + 75 + 62.5 = 145.8cm^2$

(b) (i) $\Delta$AMC: $tan\theta = \frac{1.66}{2.5}$

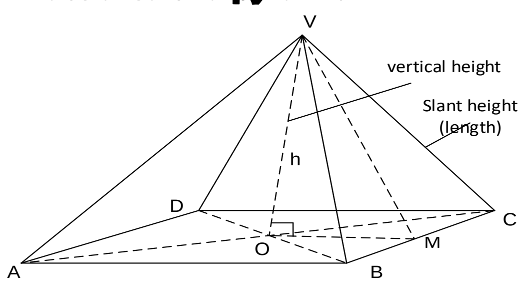
$$\theta = 33.6°$$

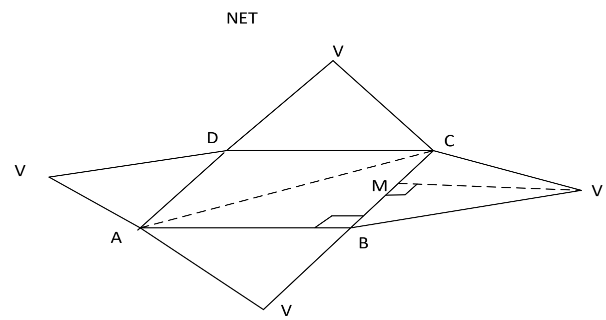
$(ii)$ $\Delta ABC$: $tan\theta = \frac{2.5}{1.66}$

$$\theta = 56.4°$$

$$< BAC = 2x56.4° = 112.8°$$

**Surface area of a pyramid**



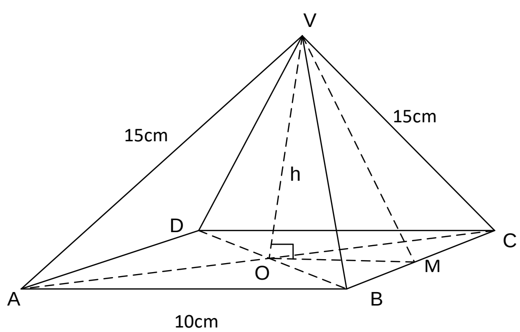



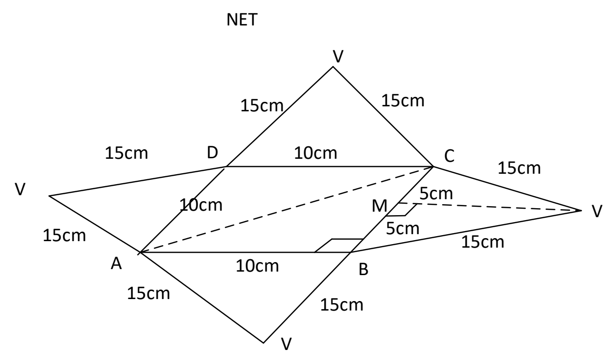

$$\boxed{T.S.A\ of\ a\ pyramid = area\ of\ the\ slant\ faces + area\ of\ the\ base}$$

**Examples**

1. Find the surface area of a right pyramid with a square base of side 10cm and slant height 15cm

**Solution**




For $\Delta VBC$: $MV = \sqrt{15^2 - 5^2} = \sqrt{200} = 14.14cm$

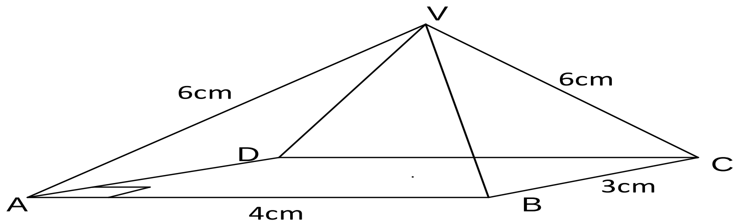
Area of 4$\Delta = 4x\frac{1}{2}x14.14x10 = 282.8cm^2$

Area of base$= 10x10 = 100cm^2$

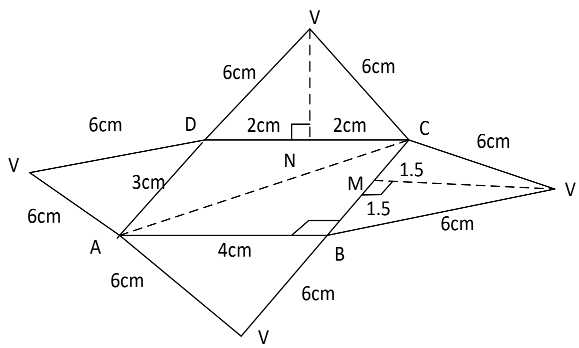
$$T.S.A\ of\ a\ pyramid = area\ of\ the\ slant\ faces + area\ of\ the\ base$$

$$T.S.A\ of\ a\ pyramid = 282.8 + 100 = 382.8cm^2$$

2. Find the total surface area of a right pyramid shown below




**Solution**




For $\Delta VBC$: $MV = \sqrt{6^2 - 1.5^2}$

$= \sqrt{33.75} = 5.809cm$

Area of 2Δ= $2x\frac{1}{2}x5.809x3 = 17.427cm^2$

For $\Delta VDC$: $NV = \sqrt{6^2 - 2^2}$

$= \sqrt{32} = 5.657cm$

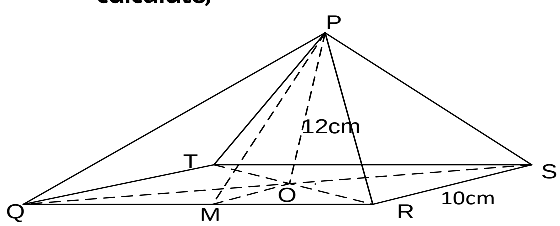
Area of 2Δ= $2x\frac{1}{2}x5.657x4 = 22.627cm^2$

Area of base= $4x3 = 12cm^2$

$T.S.A\ of\ a\ pyramid = 12 + 22.627 + 17.427$

$= 52.054cm^2$

3. The figure below shows a right pyramid standing on a square base of side 10cm. the diagonal of the square QRST meet at O and M is the midpoint of QR. Given that the vertical height of the pyramid is 12cmand that all that all the slant ages are equal, calculate;




(a) Length of PM
(b) Angle made by the planes PQR and QRST
(c) Total surface area of the pyramid

**Solution**

$\Delta PMO: PM = \sqrt{12^2 + 5^2} = 13cm$

$$tan\theta = \frac{12}{5}$$

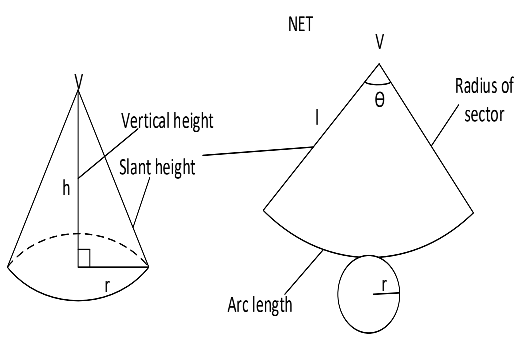
$$\theta = tan^{-1}(2.4) = 67.4°$$

Area of 4Δ= $4x\frac{1}{2}x13x10 = 260cm^2$

Area of base= $10x10 = 100cm^2$

$T.S.A\ of\ a\ pyramid = 260 + 100 = 360cm^2$

**Surface area of a cone**




circumference of a cone = arc length

$$2\pi r = \frac{\theta}{360}x2\pi l$$

$$\frac{r}{l} = \frac{\theta}{360} \ldots\ldots\ldots (i)$$

Curved S.A of a cone = Area sector

Curved S.A of a cone = $\frac{\theta}{360}x\pi l^2$

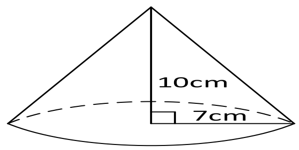
Curved S.A of a cone = $\frac{r}{l}x\pi l^2$

$$\boxed{curved\ surface\ area\ of\ a\ cone = \pi rl}$$

T.S.A of a solid cone = $\pi rl + \pi r^2$

**Examples**

1. The figure below shows a solid cone of base radius 7cm and height 10cm




Find;

(a) The slant height of the cone

(b) The total surface area of cone

**Solution**

$$l^2 = 10^2 + 7^2$$
$$l = \sqrt{149} = 12.21cm$$

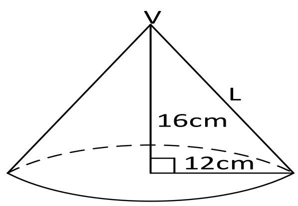
T.S.A of a cone $= \pi rl + \pi r^2$

T.S.A of a cone $= \frac{22}{7}x7x(21.21 + 7)$

$= 422.62cm^2$

2. Find the surface area of a solid cone of base radius 12cm and height 16cm. Take $\pi = \frac{22}{7}$

**Solution**



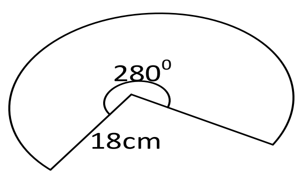

$$l^2 = 16^2 + 12^2$$
$$l = \sqrt{400} = 20cm$$

T.S.A of a cone $= \pi rl + \pi r^2$

T.S.A of a cone $= \frac{22}{7}x12x(20 + 12)$

$= 1206.86cm^2$

3. The figure below shows a sector If a circle of radius 18cm which subtends an angle of 280° at the centre of the circle.




Calculate

(i) The base radius of the cone

(ii) Total surface area of the cone

**Solution**

Curved S.A of a cone = Area sector

$$\pi rl = \frac{\theta}{360}x\pi l^2$$
$$r = \frac{280°}{360}x18 = 14cm$$

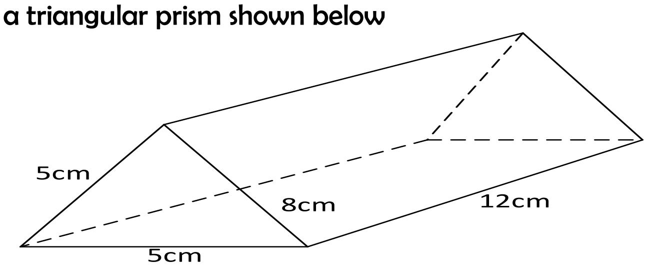
T.S.A of a cone $= \pi rl + \pi r^2$

T.S.A of a cone $= \frac{22}{7}x14(18 + 14)$

$= 1408cm^2$

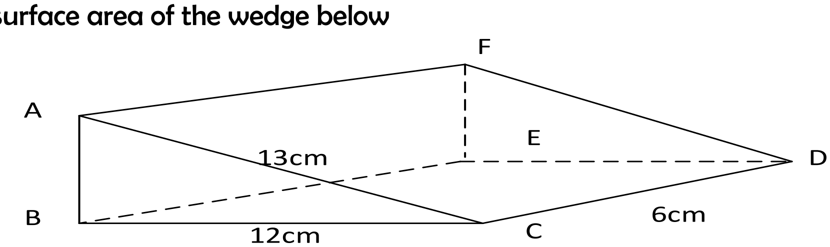
**Exercise**

1. Find the surface area of a triangular prism shown below



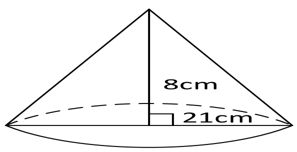

**An**($= 1188cm^2$),

2. Find the surface area of the wedge below



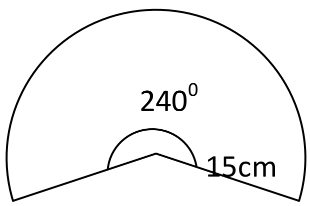
**An**($= 240cm^2$),

3. Calculate its surface area of a right pyramid whose base is a square of side 10cm and its slant side is 15cm long. **An**($= 382cm^2$),
4. Find the surface area of a cone of base radius 7cm and height 10cm. **An**($= 422.5cm^2$),
5. Find the curved surface area of a cone of base radius 5cm and slant height 21cm. **An**($= 330cm^2$),
6. Find the curved surface area of a cone of height 24cm and slant height 25cm. **An**($= 550cm^2$)
7. A sector of a circle of radius 9cm, having an angle of 80°, is bent to form a cone. Find; $\pi = 3.14$
   (a) Length of the arc of the sector
   (b) Radius of the base of the cone
   (c) Total surface area of the cone
   **An**($(a) = 12.56$ $(b) = 2cm, (c) = 69.08cm^2$)
8. The figure below shows a solid cone of base diameter 21cm and height 8cm




Find;
(a) The slant height of the cone
(b) The total surface area of cone
**An**($(a) = 13.2cm, (b) = 782.1cm^2$)

9. A sector of a circle of radius 15cm subtends an angle of 240° at the centre of the circle.

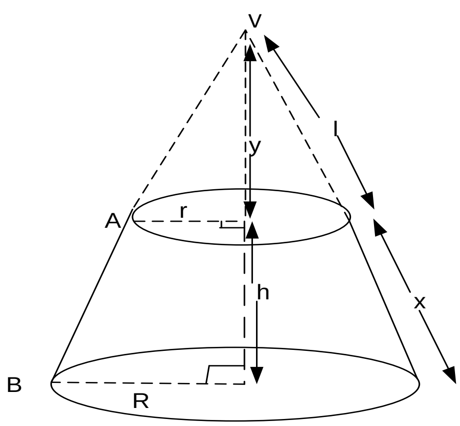


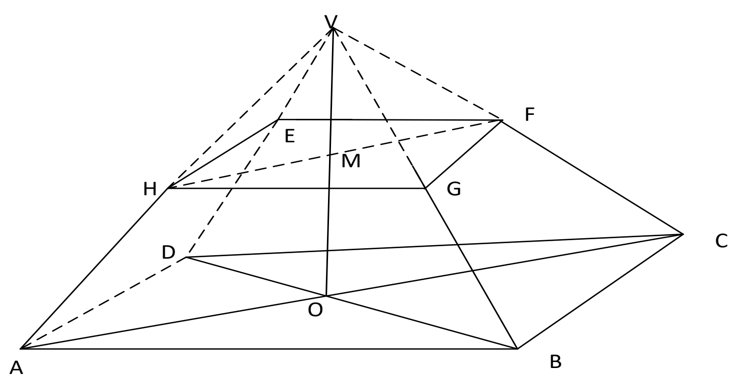

(a) Find the area of the sector
(b) The sector is cut off and folded to form a solid cone. Find the;
   (i) base radius of the cone
   (ii) Total surface area of the cone
   (take $\pi = \frac{22}{7}$)

**An**( $(a) = 471.429cm^2, (b)(i) = 10cm, (ii) = 785.5cm^2$)

**Surface area of a frustrum**

When a cone or a pyramid is cut through parallel to the base and the top part (small cone or small pyramid ) similar to the original cone or pyramid( big cone or big pyramid) is removed, the remaining part will be a solid called **a frustrum**



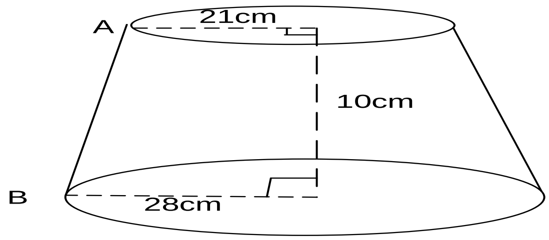




Curved S.A of frustrum= $curved\ S.A_{big\ cone} - curvved\ S.A_{small\ cone}$
$= \pi RL - \pi rl$

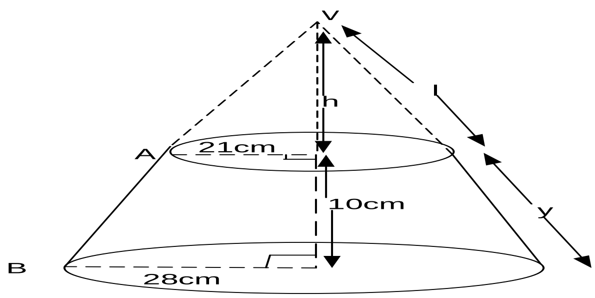
Slanting S.A of frustrum= $\left( S.A_{big\ pyramid} - S.A_{small\ pyramid} \right)$

**Examples**

1. The diagram below shows a right cone of base radius 28cm from which a small cone is cut off to form a frustrum. The top radius is 21cm and its height is 10cm. Calculate the curved surface area of the frustrum(take $\pi = \frac{22}{7}$)




**Solution**




$L:S:F = 21:28$

$h:(h+10)$

$$21h + 210 = 28h$$
$$h = 30cm$$

slant height of a small cone; $l = \sqrt{30^2 + 21^2}$

$$l = \sqrt{1341} = 36.62cm$$

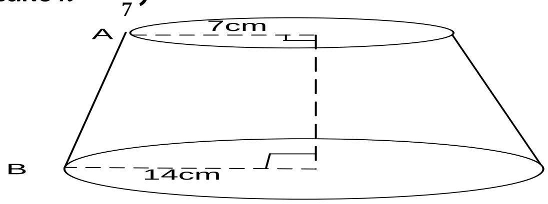
Slant height of a big cone; L= $\sqrt{(40)^2 + 28^2}$

$$L = \sqrt{2384} = 48.83cm$$

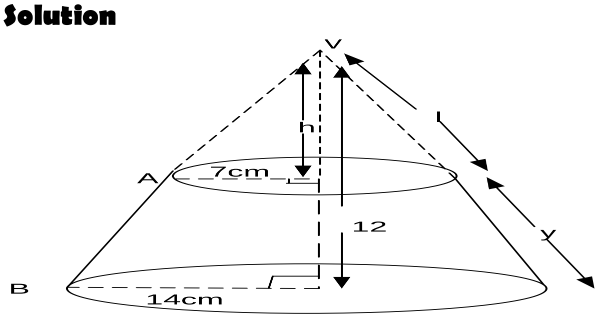
S.A of curved surface= $S.A_{big\ cone} - S.A_{small\ cone}$

$$= \pi RL - \pi rl$$
$$= \pi x28x48.83 - \pi x21x36.62$$
$$= 1880cm^2$$

2. The diagram below shows a right cone of base radius 14cm and height 12cm from which a small cone is cut off to form a frustrum. The top radius is 7cm. Calculate the curved surface area of the frustrum(take $\pi = \frac{22}{7}$)




**Solution**




$L:S:F = 7:14$

$h:12$

$$12x7 = 14h$$
$$h = 6cm$$

slant height of a small cone; $l = \sqrt{6^2 + 7^2}$

$$l = \sqrt{85} = 9.22cm$$

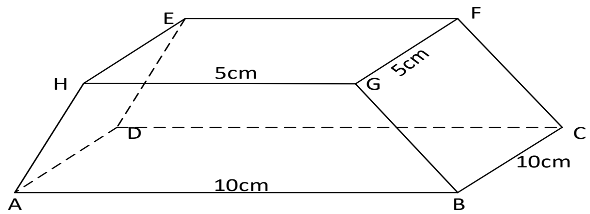
Slant height of a big cone; L= $\sqrt{(14)^2 + 12^2}$

$$L = \sqrt{340} = 18.44cm$$

S.A of curved surface= $S.A_{big\ cone} - S.A_{small\ cone}$

$$= \pi RL - \pi rl$$
$$= \pi x14x18.44 - \pi x7x9.22$$
$$= 608.35cm^2$$

3. The diagram below shows a frustrum of a pyramid made by cutting of a small pyramid halfway up the vertical height of the original pyramid.

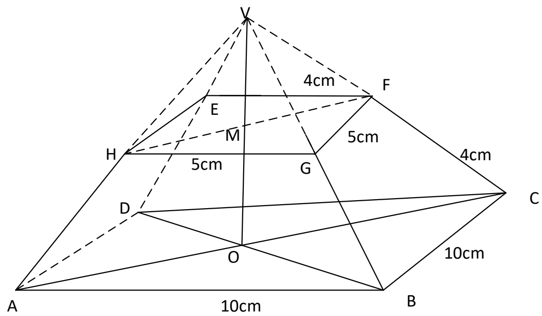



If the slant height of original pyramid is 8cm has a square base of 10cm, find;

(a) Vertical height of the original pyramid

(b) Surface area of the frustrum

**Solution faulty**




Area of $\Delta VBC$: $\frac{1}{2}x10x8 = 40cm^2$

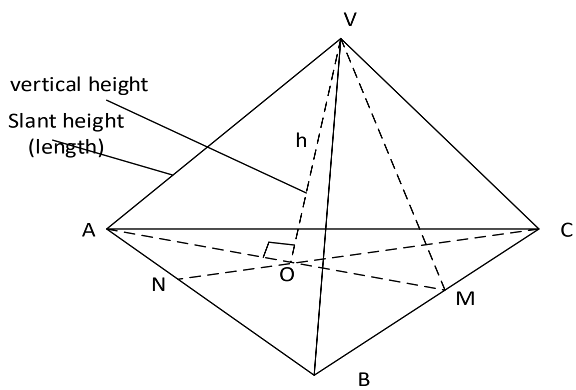
$$AC = \sqrt{676} = 26cm$$
$$OC = \frac{1}{2}xAC = \frac{1}{2}x26 = 13cm$$
$$\Delta OVC: OV^2 = 28^2 - 13^2 = 615$$
$$OV = \sqrt{615} = 24.8cm$$

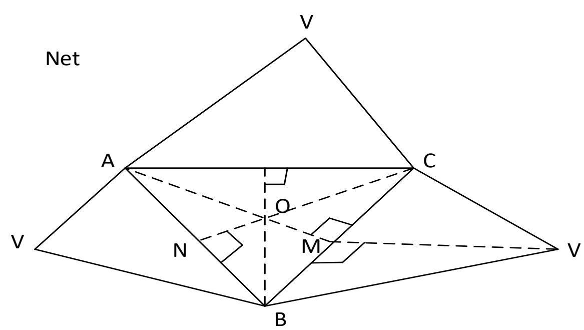
Height of small pyramid; h= $\frac{1}{2}x24.8 = 12.4cm$

Volume of frustrum= $V_{big\ pyramid} - V_{small\ pyramid}$

$$= \frac{1}{3}x24x10x24.8 - \frac{1}{3}x12x5x12.4$$
$$= 1736cm^3$$

**SURFACE AREA OF A TETRAHEDRAL PYRAMID(TETRAHEDRON)**



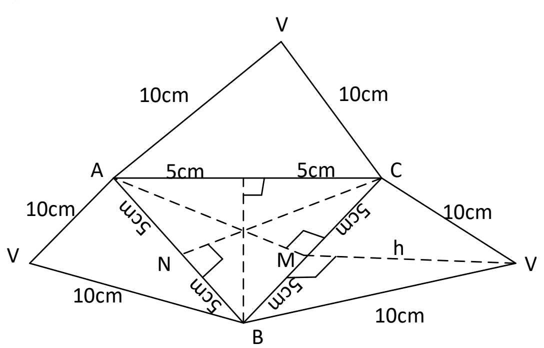




$$\boxed{T.S.A\ of\ a\ tetrahedron = area\ of\ the\ slant\ faces + area\ of\ the\ base}$$

**Example**

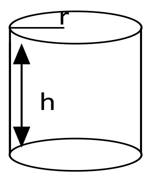
1. Calculate the surface area of the solid which is a tetrahedron whose faces are equilateral triangles of side 10cm.

**Solution**

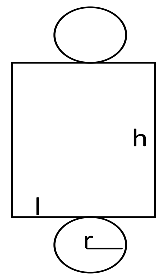



$$h = \sqrt{10^2 - 5^2} = \sqrt{75} = 8.66cm$$
$$T.S.A\ of\ a\ tetrahedron = 5x\frac{1}{2}bh$$
$$= 5x\frac{1}{2}x10x8.66 = 216.51cm^2$$

**Surface area of a cylinder**




Net




For a cylinder the length of the rectangle is equals to the circumference of the circular base
curved surface area of a cylinder=(area of a rectangle)= $lh$

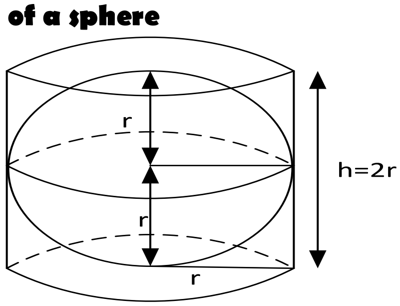
$$\boxed{Curved\ Surface\ area\ of\ a\ cylinder = 2\pi rh}$$

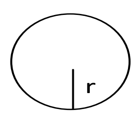
**Total Surface area of a cylinder**$= 2\pi rh + 2\pi r^2 \quad (\,closed\ at\ both\ ends)$
**Total Surface area of a cylinder**$= 2\pi rh + \pi r^2 \quad (\,closed\ at\ one\ ends)$

**Example**
Find the surface area of a cylinder of radius 1.4cm and height 10cm
**Solution**
$T.S.A = 2\pi rh + 2\pi r^2$

$$T.S.A = 2x\frac{22}{7}x1.4(10 + 1.4) = 100.32cm^2$$

**Surface area of a sphere**




Surface area of a sphere = curved surface area of a cylinder

$$= 2\pi rh = 2\pi rx2r$$

$$\boxed{surface\ area\ of\ a\ sphere = 4\pi r^2}$$

**Examples**

1. Find the surface area of a sphere of diameter 14cm.

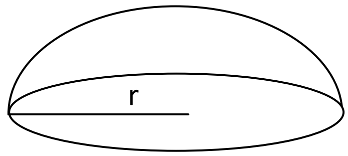
$$surface\ area\ of\ a\ sphere = 4\pi r^2 = 4x\frac{22}{7}x7^2 = 616cm^2$$

2. A sphere has a surface area of $326.2cm^2$. Find its radius
**Solution**

$$surface\ area\ of\ a\ sphere = 4\pi r^2$$
$$4x\frac{22}{7}xr^2 = 326.2$$

$$r = \sqrt{\frac{326.2x7}{4x22}} = 5.094cm$$

**Surface area of a hollow hemisphere**




Surface area of a hemisphere $= \frac{S.A\ of\ a\ sphere}{2}$

Surface area of a hemisphere $= \frac{4\pi r^2}{2}$

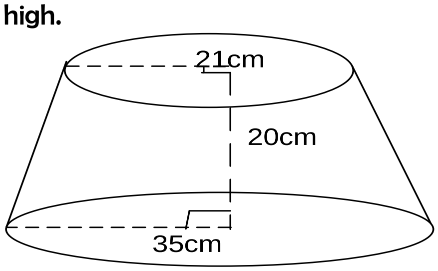
$$\boxed{S.A.of\ a\ hemisphere = 2\pi r^2}$$

Find the surface area of a solid hemisphere of radius 3.9cm.

$$surface\ area\ of\ a\ hemisphere = 2\pi r^2 + \pi r^2 = 3x\frac{22}{7}x3.9^2 = 143.35cm^2$$

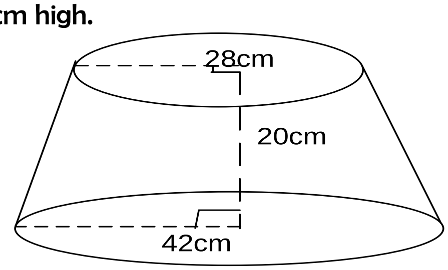
**Exercise**

1. An open cylindrical water reservoir of radius 10.5m has a curved surface area of 198m. the reservoir is to be cemented on all its inside surfaces at a cost of $sh\ 800$ per square metre. Take $\pi = \frac{22}{7}$ find,
   (a) The height of the reservoir
   (b) Cost of cementing the reservoir
   (c) **An**( $(a)h = 3m, (b)shs435{,}600)$
2. The diagram below shows a frustrum which is made by removing a small cone from the original right cone. The base radius of the original cone is 35cm and that of the removed cone is 21cm. the frustrum is 20cm high.



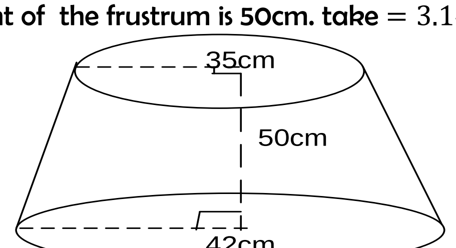

Calculate the total surface area of frustrum
**An**( $= 9530cm^2$)

3. The diagram below shows a frustrum which is made by removing a small cone from the original right cone. The base radius of the original cone is 42cm and that of the removed cone is 28cm. The frustrum is 20cm high.




Calculate the total surface area of frustrum
**An**( $= 13380cm^2$)

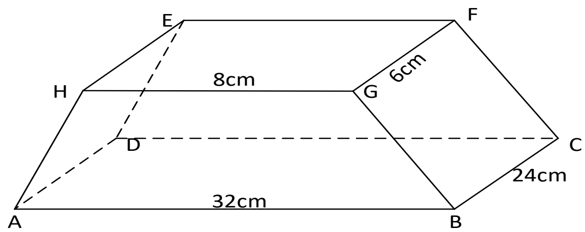
4. The diagram below shows a solid frustrum whose top radius is 35cm and bottom radius 42cm. the height of the frustrum is 50cm. take $= 3.14$




(a) Calculate
   (i) Curved surface area of he frustrum
   (ii) Total surface area of the frustrum
(b) The frustrum is to be painted on all surfaces at a cost of sh 2000 per square meter. Find the cost of painting the frustrum

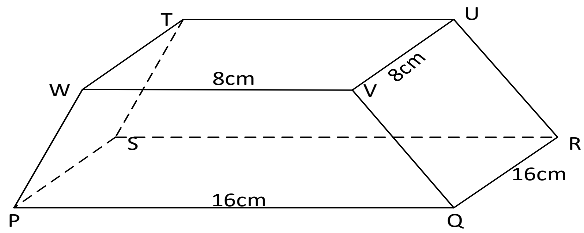
**An**( $(a)(i) = 12100cm^2, (ii) = 21500cm^2,$
$(b) = sh4{,}300)$

5. The diagram below shows a frustum of a pyramid made by cutting off a small pyramid along the plane EFGH which is parallel to the base ABCD and exactly two thirds way up the vertical height of the original pyramid.




The frustrum stands on a rectangular base measuring 32cm by 24cm while the removed pyramid stands on a rectangular base measuring 8cm by 6cm. If the slant height of original pyramid is 44cm, find Total surface area of the frustrum **An**$(= 2998cm^2)$

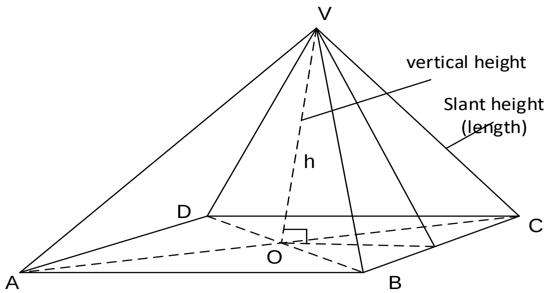
6. The diagram below shows a frustrum of a pyramid standing on a square base PQRS of side 16cm




The plane TUWV is parallel to plane PQRS. Given that $WV = VU = 8cm$ and the slant height of the original pyramid is 26cm, find the total surface area of the frustrum **An**$(= 914cm^2)$

# VOLUME OF SOLIDS

## Volume of a pyramid




$$\boxed{Vol.of\ pyramid = \frac{1}{3}base\ area\ \ x\ height\ (h)}$$

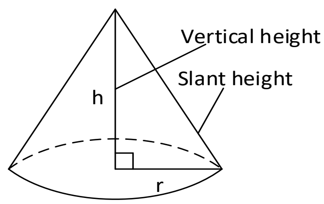
**Example**

1. Find the volume of a right pyramid whose base is a square of side 12cm and height 18cm

**Solution**

$$Vol.of\ pyramid = \frac{1}{3}base\ area\ \ x\ height\ (h) = \frac{1}{3}x12x12\ \ x\ 18 = 864cm^3$$

## Volume of a cone




$$Vol.of\ cone\ = \frac{1}{3}base\ area\ \ x\ height\ (h)$$

$$\boxed{Vol.of\ cone\ \ = \frac{1}{3}\pi r^2\ \ x\ height\ (h)}$$

**Examples**

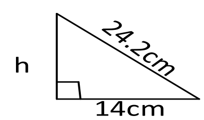
10. Find the volume of a cone when;
    (a) Radius= $7cm$ and height = $18cm$
    (b) Radius = $14cm$ and slant height = $24.2cm$

**Solution**

(a) $Vol.of\ cone\ \ = \frac{1}{3}\pi r^2\ \ xh$

$$= \frac{1}{3}x\frac{22}{7}x7^2\ \ x18 = 924cm^2$$

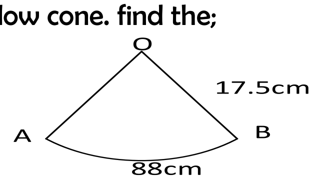
(b)




$$h = \sqrt{24.2^2 - 14^2} = 19.74cm$$

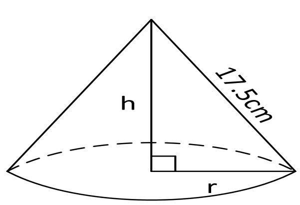
volume= $\frac{1}{3}x\frac{22}{7}x14^2\ \ x19.74 = 4053.28cm^2$

11. The figure below shows a sector of a circle radius 17.5cm and arc length 88cm. The sector is folded to form a hollow cone. find the;




(a) Radius of the cone
(b) volume of a cone

**Solution**




(iii) circumference of a cone = arc length

$$2\pi r = 88$$

$$2x\frac{22}{7}xr = 88$$

$$r = 14cm$$

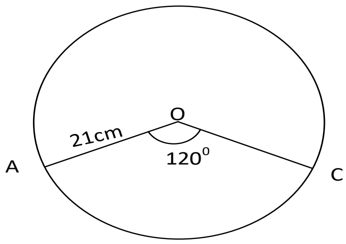
(i) $h^2 = l^2 - r^2 = 17.5^2 - 14^2$

$$h = \sqrt{110.25} = 10.5cm$$

Volume= $\frac{1}{3}\pi r^2 h = \frac{1}{3}x\frac{22}{7}(14)^2 x10.5$

$$= 2156cm^3$$

12. The figure below shows a circle centre O and radius 21cm. The minor arc ABC subtends an angle of 120° at the centre of the circle.




(a) Find the area of the minor sector

(b) The minor sector is cut off and folded to form a hollow cone. Find the;

(i) base radius of the cone

(ii) Vertical height of the cone

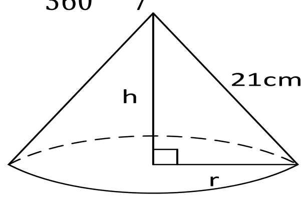
(iii) Volume of the cone

(take $\pi = \frac{22}{7}$)

**Solution**

(a) Area sector$= \frac{\theta}{360}\pi r^2$

$$= \frac{120°}{360} x \frac{22}{7}(21)^2 = 462cm^2$$




(i) Curved S.A of a cone = Area sector

$$\pi rl = 462$$

$$\frac{22}{7}x21xr = 462$$

$$r = 7cm$$

(ii) $h^2 = l^2 - r^2 = 21^2 - 7^2$

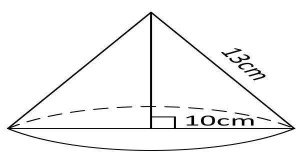
$$h = \sqrt{392} = 19.8cm$$

(iii) Volume$= \frac{1}{3}\pi r^2 h = \frac{1}{3}x\frac{22}{7}(7)^2 x19.8$

$$= 1016.4cm^3$$

**Exercise**

1. The figure below shows a solid cone of base diameter 10cm and slant height 13cm



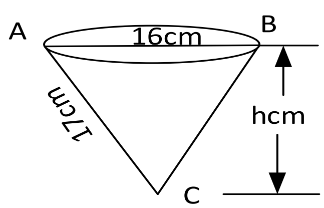

Find;

(c) The total surface area of the cone

(d) The volume the of cone

**An**($(a) = 282.6cm^2, (b) = 314cm^3$)

2. The diagram below shows a right circular cone ABC of vertical height hcm and slant side AC=BC= $17$ and base diameter AB= $16cm$.




Find;

(a) h

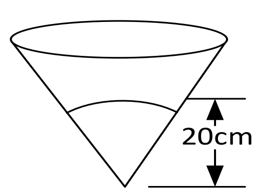
(b) The capacity of the cone

**An**( $(a) = 15cm, (b) = 1005.44cm^3,$)

3. A right circular conical flask of base radius 10cm and vertical height 30cm has a maximum internal capacity of 3 litres.

(a) Find the difference between the total volume of the flask and its internal capacity

(b) The flask is inverted such that its apex is at the bottom. It is then filled with water to a depth of 20cm




Find the radius of the water surface

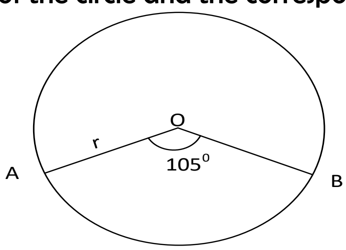
(c) The water is then poured into a rectangular trough of base 25cm by 16cm. find the depth of the water in the trough.

**An(** $(a) = 141.6cm^3, (b) = 6.67cm, (c) = 2.33cm)$

13. A sector of a circle of radius 42cm subtends an angle of 120° at the centre of the circle.
    (a) Find the;
        (i) area of the sector (ii) Length of the arc
    (b) The sector is cut off and folded to form a hollow cone. Find the;
        (i) base radius of the cone (ii) Vertical height of the cone
        (iii) Volume of the cone in litres
    (take $\pi = \frac{22}{7}$)

14. A sector of a circle of radius 40cm subtends an angle of 126° at the centre of the circle.
    (a) Find the;
        (i) area of the sector (ii) Length of the arc
    (b) The sector is cut off and folded to form a hollow cone. Find the;
        (i) base radius of the cone (ii) Vertical height of the cone
        (iii) Volume of the cone in litres
    (take $\pi = \frac{22}{7}$)
    **An(** $(a)(i) = 1760cm^2, (ii) = 88cm, (b)(i) = 14cm, (ii) = 37.5cm, (iii) = 7.7\ litres)$

15. The figure below shows a circle centre O and radius rcm. The minor arc AB subtends an angle of 105° at the centre of the circle and the corresponding sector AOB has an area of 528$cm^2$.

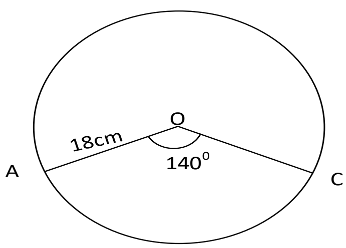



(a) Find the radius r of the circle

(b) The sector is cut off and folded to form a hollow cone. Find the;
    (i) base radius of the cone
    (ii) Vertical height of the cone
    (iii) Volume of the cone in litres
    (take $\pi = \frac{22}{7}$)

**An(** $(a)\ r = 24cm\ (b)\ (i) = 7cm, (ii) = 22.96cm, (iii) = 1179cm^3)$

16. A sector of a circle of radius 42cm subtends an angle of 120° at the centre of the circle.




(a) Find the area of the minor sector
(b) The sector is cut off and folded to form a hollow cone. Find the;
    (i) base radius of the cone
    (ii) Vertical height of the cone
    (iii) Volume of the cone
    (take $\pi = \frac{22}{7}$)

**An(** $(a) = 396cm^2, (b)(i) = 7cm, (ii) = 16.6cm, (iii) = 852cm^3)$

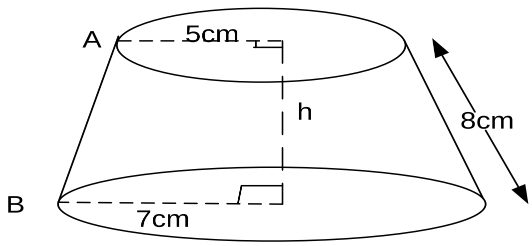
**Volume of a frustrum**

Volume of frustrum= $volume_{big\ cone} - volume_{small\ cone}$

Volume of frustrum= $\left(Volume_{big\ pyramid} - Volume_{small\ pyramid}\right)$

**Examples**

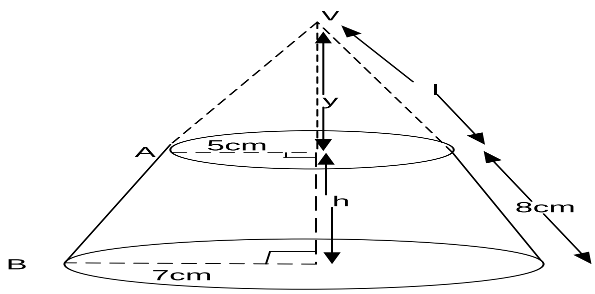
1. The figure below shows a lamp shade in the form of a conical frustrum. (take $\pi = \frac{22}{7}$)




The top and bottom radii are 5cm and 7cm respectively. The slant height AB is 8cm

(a) Calculate the volume of the lampshade
(b) Find the surface area of the curved surfaces of the lampshade
(c) Find the vertical height h of the lampshade

**Solution**




$$L.S.F = 5:7$$
$$l:(l+5)$$
$$5l + 40 = 7l$$
$$l = 20cm$$

Height of a small cone; $y = \sqrt{20^2 - 5^2} = \sqrt{375}$

$$y = 19.36cm$$

Height of a big cone; H$= y + l = \sqrt{28^2 - 7^2} = \sqrt{735}$

$$H = 27.11cm$$

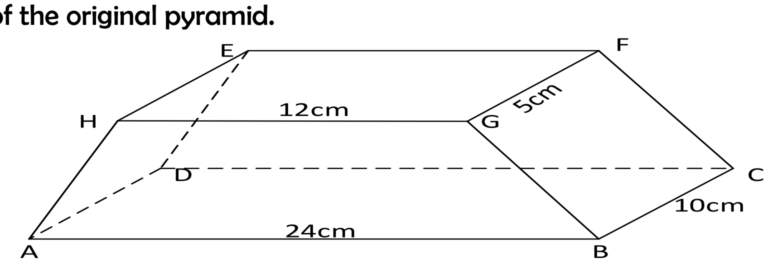
Volume of lampshade $= V_{big\ cone} - V_{small\ cone}$

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi x 7^2 x 27.11 - \frac{1}{3}\pi x 5^2 x 19.36$$
$$= 884.36cm^3$$

S.A of curved surface$= S.A_{big\ cone} - S.A_{small\ cone}$

$$= \pi RL - \pi rl$$
$$= \pi x 7x(20+8) - \pi x 5x20$$
$$= 301.59cm^2$$
$$h = 27.11 - 19.36 = 7.75cm$$

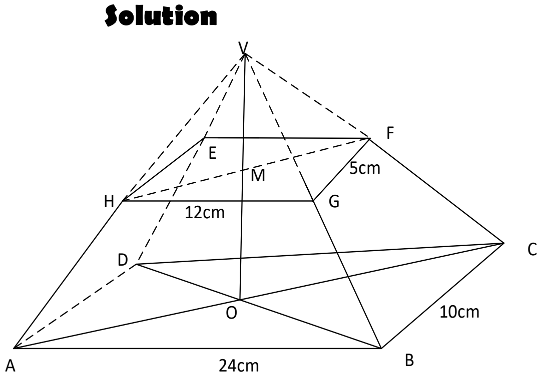
2. The diagram below shows a frustrum of a pyramid made by cutting of a small pyramid halfway up the vertical height of the original pyramid.




The frustrum stands on a rectangular base measuring 24cm by 10cm while the removed pyramid stands on a rectangular base measuring 12cm by 5cm. If the slant length of original pyramid is 28cm, find;

(c) Vertical height of the original pyramid
(d) Volume of the frustrum

**Solution**



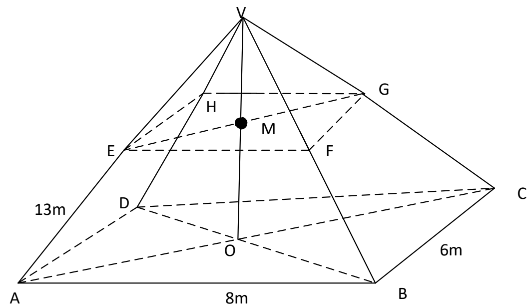

$$\Delta ABC: AC = \sqrt{24^2 + 10^2} = \sqrt{676} = 26cm$$
$$OC = \frac{1}{2} x AC = \frac{1}{2} x 26 = 13cm$$
$$\Delta OVC: OV^2 = 28^2 - 13^2 = 615$$
$$OV = \sqrt{615} = 24.8cm$$

Height of small pyramid; h$= \frac{1}{2} x 24.8 = 12.4cm$

Volume of frustrum$= V_{big\ pyramid} - V_{small\ pyramid}$

$$= \frac{1}{3} x 24 x 10 x 24.8 - \frac{1}{3} x 12 x 5 x 12.4$$
$$= 1736cm^3$$

3. In the diagram below VABCD is a pyramid with a rectangular base ABCD and V, the vertex O is the center of the base ABCD




$AB = 8m$, $BC = 6m$, $VC = VB = VA = VD = 13m$. M is a point on VO such that $3MV = OV$. M is also the centre of the base $EFGH$ of small pyramid VEFGH similar to VABCD which is to be cut off from the original pyramid VABCD. Find the;

(i) Dimensions of the base EFGH

(ii) Height of pyramid VABCD

(iii) Volume of the remaining part of a pyramid VABCD when VEFGH is cut off

**Solution**

(i) $MV : OV = 1 : 3$

$$FG : 6$$
$$3FG = 6$$
$$FG = 2m$$

$MV : OV = 1 : 3$

$$EF : 8$$
$$3EF = 8$$
$$EF = 2.67m$$

(ii) $\Delta ABC: AC = \sqrt{8^2 + 6^2} = \sqrt{100}$

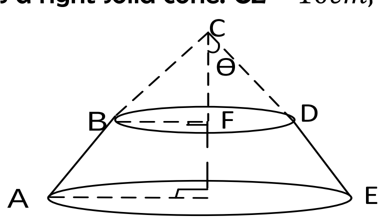
$$= 10m$$
$$OC = \frac{1}{2}xAC = \frac{1}{2}x10 = 5m$$
$$\Delta OVC: OV^2 = 13^2 - 5^2 = 144$$
$$OV = \sqrt{144} = 12m$$

(iii) Volume of frustrum$= V_{big\ pyramid} - V_{small\ pyramid}$

$$= \frac{1}{3}x8x6x12 - \frac{1}{3}x2x2.67x4$$
$$= 184.88m^3$$

**Exercise**

1. ABCDE is a right solid cone. CE$= 10cm$, $\theta = 30°$, $CD : DE = 2 : 3$. The cone BCD was cut off



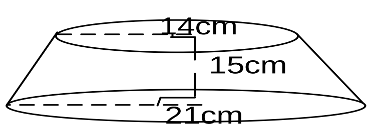

Find

(i) Total surface area of the remaining portion ABDE

(ii) Volume of the cone BCD

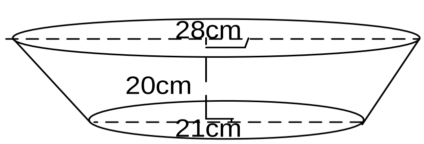
**An**( $(i) = 210.514cm^2, (ii) = 14.52cm^3$)

2. The diagram shows a solid frustrum with base radius 21cm and top radius 14cm. the frustrum is 15cm high .




Calculate the volume of the metal used in the frustrum **An**($= 14630cm^3$)

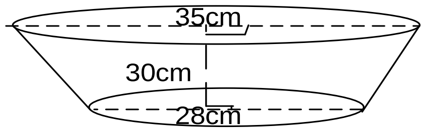
3. The diagram below shows a frustrum which represents a bucket with an open end diameter of 28cm and bottom diameter of 21cm. The bucket is 20cm deep, take $\pi = \frac{22}{7}$




Calculate the capacity of the bucket in litres

**An**($= 9.5\ litres$)

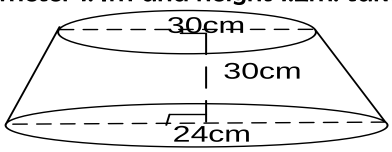
4. The diagram below shows a frustrum which represents a bucket with an open end diameter of 35cm and bottom diameter of 28cm. The bucket is 30cm deep and its used to fill an empty cylindrical tank of diameter 2.8m and height 1.5m. take $\pi = \frac{22}{7}$




Calculate number of buckets that must be draw in order to fill the tank

**An**$(=394)$

5. The diagram below shows a frustrum which represents a bucket with an open end diameter of 30cm and bottom diameter of 24cm. The bucket is 30cm deep and its used to fill an empty cylindrical tank of diameter 1.4m and height 1.2m. take $=3.14$

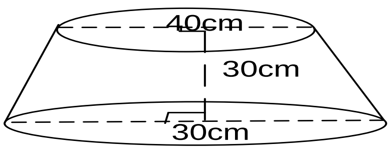



Calculate

(a) The capacity of the bucket in litres
(b) Capacity of the tank in litres
(c) Number of buckets that must be draw in order to fill the tank

**An**$((a)=17.239l, (b)=1846.32l,\ (c)=108)$

6. The diagram below shows a frustrum which represents a bucket with an open end diameter of 40cm and bottom diameter of 30cm. The bucket is 30cm deep and its used to fill an empty cylindrical tank of radius 1.2m and height 1.35m. take $\pi=\frac{22}{7}$

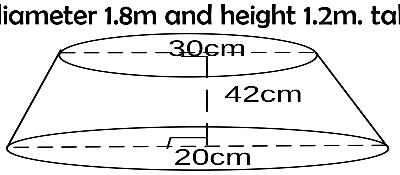



Calculate

(a) The capacity of the bucket in litres
(b) Capacity of the tank in litres
(c) Number of buckets that must be draw in order to fill the tank

**An**$((a)=29.071l, (b)=2443.886l,\ (c)=85)$

7. The diagram below shows a frustrum which represents a bucket with an open end diameter of 30cm and bottom diameter of 20cm. The bucket is 42cm deep and its used to fill an empty cylindrical tank of diameter 1.8m and height 1.2m. take $=3.14$

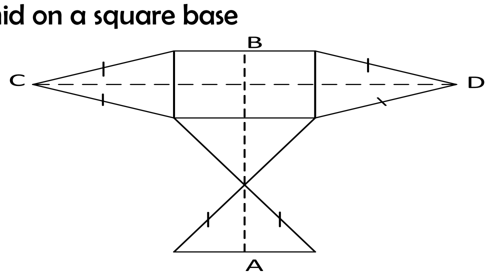



Calculate

(a) The capacity of the bucket in litres
(b) Capacity of the tank in litres
(c) Number of buckets that must be draw in order to fill the tank

**An**$((a)=20.881l, (b)=3052.08l,\ (c)=147)$

8. The diagram shows a square of side 12 cm and four congruent isosceles triangles, representing the net of a pyramid on a square base

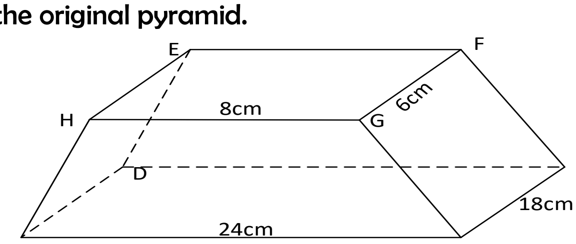



Given that $AB=CD=40cm$, calculate the

(a) (i) height of the vertex of the pyramid from the square base,
$(ii)$ Angle between a triangular face and the base of the pyramid
$(iii)$ Volume If the pyramid
(b) If the pyramid is cut horizontally at a vertical height of 2.6cm from the square base, and the upper part of the pyramid containing the vertex is thrown away, what volume remains

**An**$((i)=12.65cm, (ii)=64.6°,\ (iii)=607.2cm^3, (b)=302.7cm^3)$

9. The diagram below shows a frustrum of a pyramid made by cutting off a small pyramid along the plane EFGH which is parallel to the base ABCD and exactly two thirds way up the vertical height of the original pyramid.

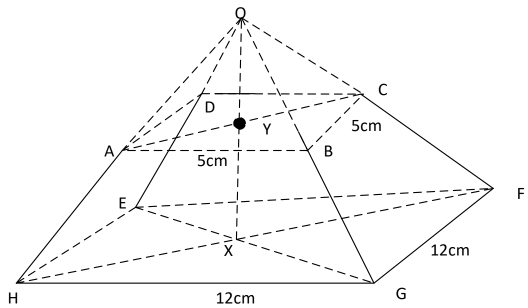



The frustrum stands on a rectangular base measuring 24cm by 18cm while the removed pyramid stands on a rectangular base measuring 8cm by 6cm. If the slant height of original pyramid is 36cm, find;

(a) Vertical height of the original pyramid
(b) Volume of the frustrum

**An**$((a)=32.7cm, (b)=4535cm^3)$

10. The diagram below shows a frustrum ABCDEFGH of a pyramid made by cutting off a small pyramid OABCD from the right pyramid OEFGH on a plane parallel to the base. The base and the top of the frustrum are squares of side 12cm and 5cm respectively.




Given that $OB = 6cm$ and each of the slant edges of the frustrum is 15cm long. Find the;

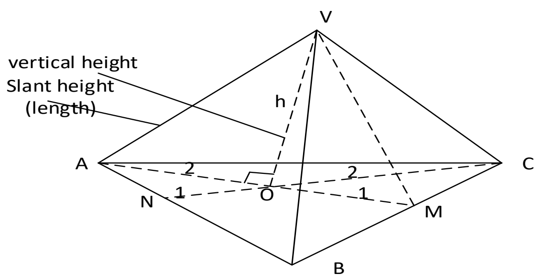
(i) Height OY of the small pyramid

(ii) Vertical Height of XY of the frustrum

(iii) Volume of the frustrum

**An(** $(a) = 4.8cm, (b) = 14.4cm\ (c) = 881.6cm^3$)

**VOLUME OF A TETRAHEDRAL PYRAMID**

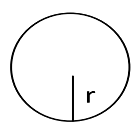



$$\boxed{Vol.of\ pyramid = \frac{1}{3}base\ area\ \ x\ height\ (h)}$$

$OC = \frac{2}{3}NC$ and $OA = \frac{2}{3}$

$MA\ NO = \frac{1}{3}NC$ and $MO = \frac{1}{3}MA$

**volume of a sphere**




$$\boxed{Vol.of\ a\ sphere = \frac{4}{3}\pi r^3}$$

**Example**

1. A sphere of radius is melted and recast into a cone. If its height is 15cm, find the radius of its base.

**Solution**

$$Vol.of\ a\ sphere = \frac{4}{3}\pi r^3$$

$$Vol.of\ a\ sphere = \frac{4}{3}x\frac{22}{7}x24^3$$

$$= 57929.143cm^3$$

$$Vol.of\ cone\ = \frac{1}{3}\pi r^2\ xh$$

$$57929.143cm^3\ = \frac{1}{3}x\frac{22}{7}xr^2\ x15$$

$$r = \sqrt{\frac{57929.143x3x7}{22x15}} = 60.716cm$$

2. The volume of a sphere is a third that of a cone. Given that the radius of a cone of height 10cm is 4.2cm. find the volume of the sphere

**Solution**

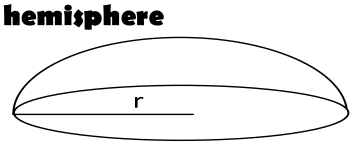
$$Vol.of\ a\ sphere = \frac{1}{3}volume\ of\ a\ cone$$

$$Vol.of\ sphere\ = \frac{1}{3}\left(\frac{1}{3}\pi r^2\ xh\right)$$

$$Vol.of\ a\ sphere = \frac{1}{3}\left(\frac{1}{3}x\frac{22}{7}x4.2^2x10\right)$$

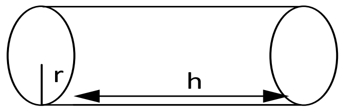
$$= 61.6cm^3$$

**Volume of a hemisphere**



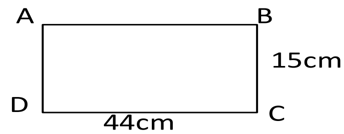

$$\boxed{Vol.of\ a\ hemisphere = \frac{2}{3}\pi r^3}$$

**Volume of a cylinder**



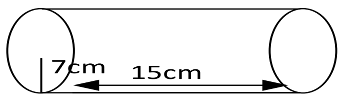

$\boxed{Volume\ of\ cylinder = \pi r^2 h}$

1. The diagram below shows a rectangle ABCD of length 44cm and width 15cm




If it is folded such that AD and BC come together to form a hollow cylindrical figure. Find the volume of the cylindrical figure formed.

**Solution**




$circumference = 2\pi r = 44$

$$r = \frac{44}{2\pi} = 7cm$$

$$Volume\ of\ cylinder = \pi r^2 h = \frac{22}{7} x 7^2 x 15$$

$$= 2310cm^3$$

2. A cylindrical tank whose diameter is 1.4 meters and height 80cm is initially empty. Water whose volume is 492.8 litres is poured into the tank. Determine the fraction of the tank filled with water.(take $\pi = \frac{22}{7}$)

**Solution**

Volume of tank= $\pi r^2 h = \frac{22}{7}(70cm)^2 x 80cm = 1{,}232{,}000cm^3$

Fraction filled= $\frac{492.8x1000\ cm^2}{1232000} = \frac{2}{5}$

3. A cylindrical tank whose diameter is 42cm and height 30cm is one third full of water. The tank is filled using a small cylindrical container of radius 4.2cm and height 10cm. find the number of full containers required to fill the tank.

**Solution**

Volume of tank= $\pi R^2 h = \pi(21)^2 x 30 = 13230\pi cm^3$

Volume of remaining water= $\frac{2}{3} x 13230\pi cm^3 = 8820\pi cm^3$

Volume of container= $\pi r^2 h = \pi(4.2)^2 x 10cm = 176.4\pi cm^3$

Number of containers= $\frac{8820\pi cm^3}{176.4\pi cm^3} = 50$

4. A rectangular container measuring 1.2m long, 70cm wide and 55cm high is half full of water. All this water is poured into an empty cylindrical tank of diameter 1.4m. Find the height to which the water rises in cm. (take $\pi = \frac{22}{7}$)

**Solution**

Volume of water $= \frac{1}{2} x 120cm x 70cm x 55cm$

$= 231000cm^3$

$\pi r^2 h = 231000$

$$\frac{22}{7} x (70)^2 x h = 231000$$

$$h = 15cm$$

5. A closed cylindrical tank of diameter 7m has a total surface area of $110m^2$. The tank which is initially one-third full of water is filled by a pump which pumps water at the rate of 2.5 litres per second. (take $\pi = 3.14$)

   (a) Find;

   (i) Height of the tank to 1 decimal place

   (ii) Volume of water required to fill the tank in litres

   (iii) Time in hours taken to fill the tank

(b) Starting with the full tank, a school uses water from this tank at the rate of 2,400 litres per day. Find how many complete days it takes to use all the water from the tank, assuming that no more water is added.

**Solution**

(i) $T.S.A = 2\pi rh + 2\pi r^2$

$110 = 2x3.14x3.5(h + 3.5)$

$h + 3.5 = 5.0045$

$h = 1.5m$

(ii) volume required= $\frac{2}{3}\pi r^2 h$

$= \frac{2}{3}x3.14x3.5^2x1.5 = 38.465m^3$

$= 38.465x1000 = 38465litres$

(iii) $2.5l = 1second$

$38465l = t\ second$

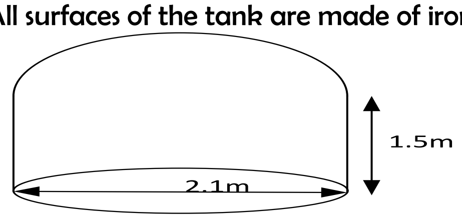
$t = \frac{38465}{2.5} = 15386s = \frac{15386}{3600} = 4.274h$

(iv) $\frac{2}{3}V = 38465l$

$V = 57697.5l$

Number of days= $\frac{57697.5}{2400} = 24\ days$

6. The figure below shows a water tank which consists of a hemispherical part surmounted on top of a cylindrical part. The two parts have the same diameter of 2.1m and the cylindrical part is 1.5m high as shown. All surfaces of the tank are made of iron sheets.




(a) Calculate the area of the iron sheet required

(b) Irons sheets cost 4000 per square meter while labour and other materials amount to 50% of the cost of iron sheets. Find the cost of constructing the tank

**(c)** Calculate the volume of the tank in litres (take $\pi = 3.14$)

**Solution**

**(a)** $T.S.A = \pi r^2 + 2\pi rh + 2\pi r^2 = \pi r(3r + 2h)$

$= 3.14x1.05x(3x1.05 + 2x1.5)$

$= 20.277m^2$

(b) Total cost= $4000x20.277 + \frac{50}{100}x4000x20.277$

$= 121{,}662/=$

**(c)** $volume = \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{2}{3}r\right)$

$= 3.14x1.05^2\left(1.5 + \frac{2}{3}x1.05\right)$

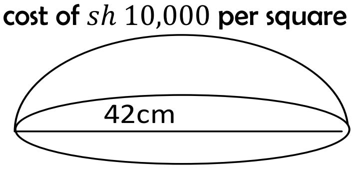
$= 7.616m^2 = 7.616x1000l$

$= 7616l$

**Exercise**

1. A rectangular water tank measuring 2.5m long, 1.8m wide and 1.2m high is half full of water. All this water is poured into an empty cylindrical tank of diameter 2.8m. calculate the height to which the water rises. **An**(43.8cm)
2. A cylindrical tank of diameter 1.4m and height 1.2m is two-thirds full of water. The tank is filled using a cylindrical bucket of diameter 35cm and height 20cm. Find the number of buckets required to fill the tank. **An**(32)
3. A cylindrical tank of radius 2m and height 1.5m initially contains water to a depth of 50cm. water is added to the tank at a rate of 62.84 litres per minute for 15 minutes. Find the new height of the water in the tank Take $\pi = 3.142$ **An**(80cm)
4. The curved surface area of a cylindrical container is 1980cm. if the radius of the container is 21cm, calculate the capacity pf the container in litres. Take $\pi = \frac{22}{7}$ **An**(20.8litres)
5. A solid metal sphere of radius 7.5cm is melted down and recast into a solid cylinder of height 15cm. In the process 4% of the metal is lost. Take $\pi = 3.14$
   (a) Find the volume of the metal used to make the cylinder
   (b) What is the radius of the cylinder. **An**( $(a) = 1695.6cm^3, (b) = 6cm,$)
6. A cylindrical tank of diameter 1.4m and height 1.2m is one-quarter full of water. This water is transferred to an empty rectangular container measuring 1.2m long and 70cm wide. Find the height of
7. the water in the container **An**(55cm)

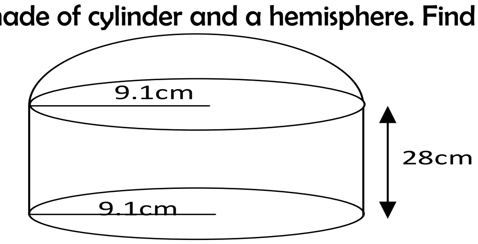
8. A cylindrical container of diameter 35cm and height 20cm is two- fifths full of water. The container is filled using a smaller cylindrical can of diameter 7cm and height 10cm. find the number of cans that must be drawn in order to fill the container **An**(30)
9. A cylindrical tank of radius 2.5m and height 1.6m initially contains water to a depth f 20cm. water is pumped into the tank at the rate of 1.35 litres per second. Find the time it takes to fill the tank **An**(5h 40min)
10. A closed cylindrical tank of diameter 4.2m has a total surface area of $46.2m^2$. The tank which is initially half full of water is filled by a pump which pumps water at the rate of 1.25 litres per second. (take $\pi = \frac{22}{7}$)
    (a) Find;
        (i) Height of the tank
        (ii) Volume of water required to fill the tank in litres
        (iii) Time in hours taken to fill the tank
    (b) Starting with the full tank, a school uses water from this tank at the rate of $1125$ litres per day. Find how many complete days it takes to use all the water from the tank, assuming that no more water is added.
    **An**( $(a)(i) = 1.4m, (ii) = \ 9702\ litres, (iii) = 2h\ 9min, (b) = 17days$)
11. The diagram below shows a solid hemispherical dome of diameter 42cm. the dome is painted on all faces at a cost of $sh\ 10{,}000$ per square meter




Calculate;
(a) total surface are of the dome
(b) Cost of painting the dome
(c) Volume of the material making the dome

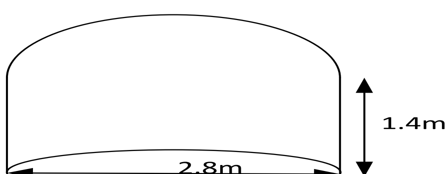
**An**( $(a) = \ 4158cm^2, (b) = sh4{,}158, (c) = 19404cm^3$)

12. The figure below shows a solid made of cylinder and a hemisphere. Find its volume




**An**( $8866.19\ cm^3$)

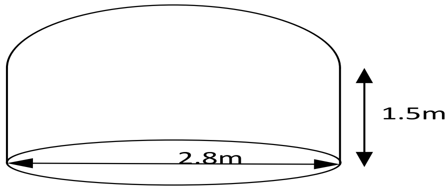
13. The figure below shows a water tank which consists of a hemispherical part surmounted on top of a cylindrical part. The two parts have the same diameter of 2.8m and the cylindrical part is 1.4m high as shown.




(a) Calculate the total surface area of the tank
**(b)** Calculate the volume of the tank in litres
**(c)** Starting with a full tank a family uses water from this tank at the rate of 185 litres per day for the first two days. After that the family uses water at the rate of 200 litres per day. Assuming that no more water is added, determine how long it takes the family to use all the water from the tank.
(take $\pi = \frac{22}{7}$)

**An**( $(a) = 30.8m^2, (b) = \ 14370\ litres, (c) = 72days$)

14. The figure below shows a water tank which consists of a hemispherical part surmounted on top of a cylindrical part. The two parts have the same diameter of 2.8m and the cylindrical part is 1.5m high as shown.



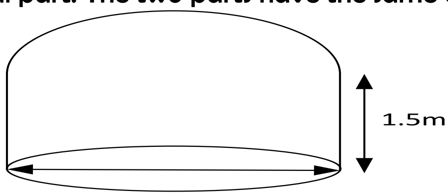

(a) Calculate the total surface area of the tank

(b) Find the cost of painting the tank at the rate of $sh\ 300$ per square meter

**(c)** Calculate the volume of the tank in litres (take $\pi = \frac{22}{7}$)

**An**( $(a) = 31.68m^2, (b) = sh\ 9504, (c) = 14990 litre$)

15. The figure below shows a water tank which consists of a hemispherical part surmounted on top of a cylindrical part. The two parts have the same diameter and the cylindrical part is 1.5m high as shown.

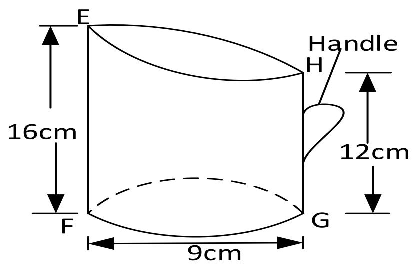



(a) Given that the total surface area of the tank is $528m^2$, find the radius of the tank

**(b)** Calculate the volume of the tank (take $\pi = \frac{22}{7}$)

**An**( $(a) r = 7m, (b) = 949.7m^3$)

**16.** The diagram below shows a container EFGH ( part of a cylindrical can) used by a shop keeper for scooping out sugar from a sack




Calculate the;

(a) Maximum volume of the sugar the container can scoop

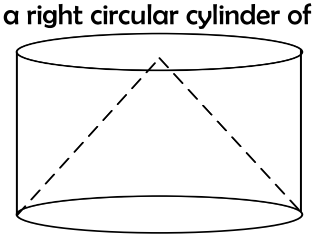
(b) Ratio of the volume of the cut-off piece of the cylindrical container to that of the container EFGH.

**An**( $(i) = 890.64cm^3, (ii) = 1:7$)

17. Three solids, a sphere, a right cone and a right cylinder are of equal surface are and the radii of their circular sections are also equal. Given that the volume of the sphere is $288\pi cm^3$, find the
   (a) The radius of the sphere
   (b) Height of the cylinder and hence determine its volume
   (c) Length of the slat side of the cone and hence determine its volume

**An**( $(a) r = 4.1cm, (b) h = 4.1cm, v = 216.55cm^3, (c) l = 16.4cm, v = 279.56cm^3$)

18. The figure above shows a trough for watering poultry. It consists of a solid cone of height 14cm, welded onto the base of a right circular cylinder of the same height. Given that the radius is 25cm at the base



(a) The volume of the cylinder

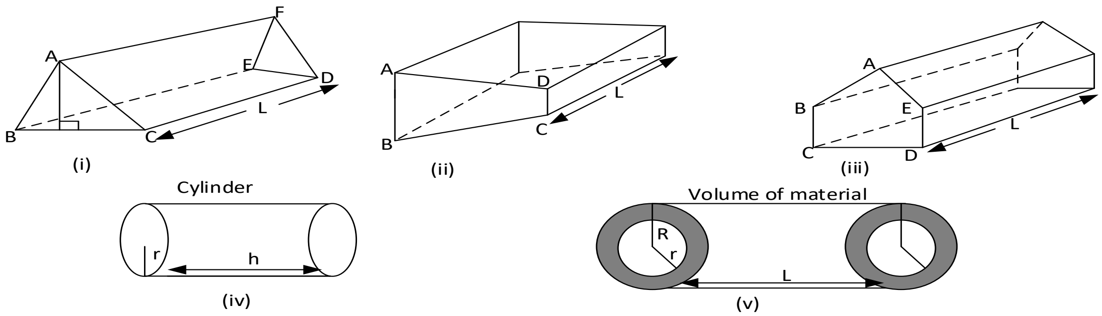
(b) The capacity of the trough when it is full

Take $\pi = 3.14$

**An**( $(a) = 27500cm^3, (b) = 18341.67cm^3$)

**PRISMS**

$$\boxed{Volume\ of\ a\ prism = \ area\ of\ cross - section\ x\ length}$$




$(i)$ $Volume\ of\ a\ prism = \ area\ of\ \Delta ABC\ x\ length$

$(ii)$ $Volume\ of\ a\ prism = \ area\ of\ trapezium\ ABCD\ x\ length$

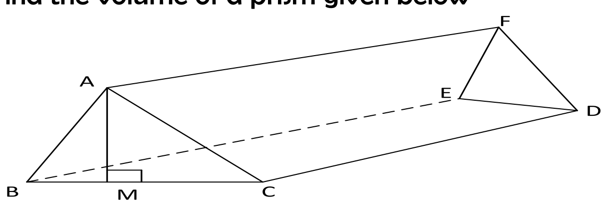
$(iii)$ $Volume\ of\ a\ prism = \ area\ of\ pentagon\ ABCDE\ x\ length$

$(iv)$ $Volume\ of\ cylinder = \ \pi r^2 h$

$(v)$ $Volume\ of\ material = \ (\pi R^2 - \pi r^2)xl$

**Examples**

1. Find the volume of a prism given below



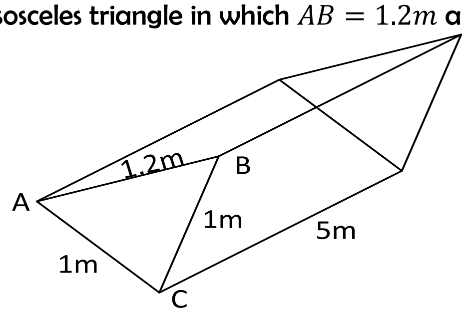

$AM = 10cm$, $BC = 15cm$ and $CD = 25cm$

**Solution**

$Vol.\,of\ a\ prism = \ area\ of\ cross - section\ x\ length$

$$= \frac{1}{2}\ (15x10)x25 = 1875cm^3$$

2. The diagram below shows a trough which has a triangular cross-section and is 5m long. Triangle ABC is an isosceles triangle in which $AB = 1.2m$ and $AC = BC = 1m$




(a) Calculate the capacity of the trough in litres

(b) The trough is $\frac{1}{4}$ fulll of water

(i) How many litres are required to fill the trough

(ii) Water is poured in the trough at the rate of 0.75 litres per second, find the time in minutes it takes to fill the trough

(c) Starting with the full trough, water is drawn from the trough using a cylindrical bucket of radius 14cm and height 25cm. how many buckets are drawn to empty the trough.

**Solution**

(a) $s = \frac{a+b+c}{2} = \frac{1.2+1+1}{2} = 1.6m$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{1.6(1.6-1.2)(1.6-1)(1.6-1)}$$

$$A = 0.48m^2$$

$Vol.\,of\ a\ trough = \ area\ of\ C.S\ x\ length$

$$= 0.48x5 = 2.4m^3$$

Capacity $= 2.4x1000 = 2400 litres$

(b) $(i)$ $required\ fillig\ volume\ = \frac{3}{4}x2400$

$$= 1800\ litres$$

$(ii)$time$= \frac{1800}{0.75} = 2400\ s = \frac{2400}{60}\ min$

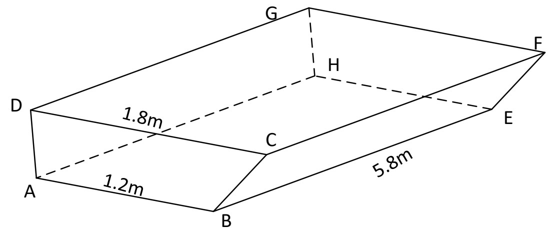
$$time = 40\ min$$

Vol of bucket$= \pi r^2 h = \frac{22}{7}x0.14^2x0.25$

$$= 0.0154m^3$$

Number of buckets$= \frac{2.4}{0.0154} = 156$

3. The diagram below shows a trough whose ABCD is a trapezium with AB being parallel to DC. DC$=$ $1.8m$, BCE$=$ $5.8m$ long as shown.




(a) Given that the trough is 40cm deep, find;

(i) Cross-sectional area of the trough in square meters
(ii) Capacity of the trough in cubic meters

(b) A second trough which is similar in shape to the trough in (a) above has a capacity of 435 litres. Calculate;

(i) The length of the second trough
(ii) The cross-sectional area of this trough

**Solution**

(a) Area of cross-section=Area of trapezium

$$=\frac{1}{2}(1.8+1.2)0.4=0.6m^2$$

volume of trough $=$ cross $-$ sectional area $xl$

$$=0.6m^2x5.8m=3.48m^3$$

(b) V.S.F= $\frac{3.48x1000l}{435l}=8$

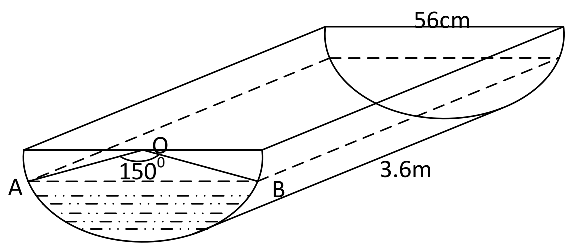
L.S.F= $\sqrt[3]{8}=2$

Length of 2nd trough= $\frac{5.2}{2}=2.9m$

A.S.F= $2^2=4$

Area of 2nd trough= $\frac{0.6}{4}=0.15m^2$

4. The diagram below shows a trough made from cutting a huge drum exactly into two. The cross-section of the trough is a semi-circle of diameter 56cm and it is 3.6m long. The trough contains water to a level which subtends an angle of 150 at the center as shown. Take $\pi=3.14$




Calculate;

(a) Volume of the water (in litres) of the water in the trough
(b) Capacity of the trough in litres

**Solution**

(a) Area of sector AOB: $A=\frac{150}{360}x3.14x28^2=1025.733\ cm^2$,

Area of triangle AOB: A= $\frac{1}{2}x28^2 sin150=196cm^2$

Area of Cross-section of water: A= $1025.733\ -196=829.733\ cm^2$

Volume of the water= $829.733\ x360=298703.88cm^3=\frac{298703.88}{1000}=298.704\ litres$

(b) Area of cross-section: $A=\frac{1}{2}x3.14x28^2=1230.88\ cm^2$,

Volume of the trough= $1230.88\ x360=443116.8cm^3=\frac{443116.8}{1000}=443.117\ litres$

1. Find the volume used to make a long cylindrical pipe 6m long whose external and internal radii are 63cm and 56cm.

**Solution**

$$Volume\ of\ material=\ area\ of\ cross-section\ x\ length$$
$$=(\pi R^2-\pi r^2)xl$$
$$=\frac{22}{7}x(63^2-56^2)x600=1{,}570{,}800cm^3$$

2. An open cylindrical reservoir of internal diameter 42m and height 4m is constructed using reinforced concrete for walls and foundation. The walls are 1.4m thick while the foundation is laid to a depth of 2m.

(a) Calculate;

(i) volume of the concrete used to construct the reservoir
(ii) The capacity of the reservoir

(b) Starting with the full reservoir, water is consumed at the rate of $25200$ litres per day. Assuming no more water is added, calculate the number of days it takes to use up all the water in the reservoir

**Solution**

(a) $(i)\ r = \frac{42}{2} = 21m$

$R = 21 + 1.4 = 22.4m$

$Vol. of\ concrete = \ area\ of\ C.S\ x\ length$

$= (\pi R^2 - \pi r^2)xl$

$= \frac{22}{7}x(22.4^2 - 21^2)x4 = 763.84m^3$

$volume\ of\ foundation = \pi R^2 h$

$= \frac{22}{7}x22.4^2 x2 = 3153.92m^3$

Total volume of concrete$= 763.84 + 3153.92$

$= 3917.76m^3$

$(ii)\quad volume\ of\ reservoir = \pi r^2 l$

$= \frac{22}{7}x21^2 x4 = 5544m^3$

**(b)** $volume\ of\ reservoir = 5544x1000\ litres$

Number of days$= \frac{5544x1000}{25200} = 220\ days$

3. A hollow metal pipe whose internal and external diameters are 5.0cm and5.6cm respectively is 2.4m long.
   (a) Find the volume of the metal in the pipe
   (b) If the pipe is melted down and cast into a solid sphere, find the radius of the sphere obtained
   (c) If the pipe is melted down and cast into a solid cone of height 20cm and in the process 5% of the metal is lost, find the radius of the cone obtained. (take $\pi = \frac{22}{7}$)

**Solution**

(a) Area of cross-section $= A_{big\ circle} - A_{small\ circle}$

$= \pi R^2 - \pi r^2 = \frac{22}{7}x2.8^2 - \frac{22}{7}x2.5^2 = 5cm^2$

volume of metal $=$ cross $-$ sectional area $xl$

$= 5cm^2 x240cm = 1200cm^3$

(b) Volume of sphere $= \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi r^3 = 1200$

$r = \sqrt[3]{\frac{1200x3}{4\pi}} = 6.59cm$

(c) Volume of remaining metal$= \frac{95}{100}x1200cm^3$

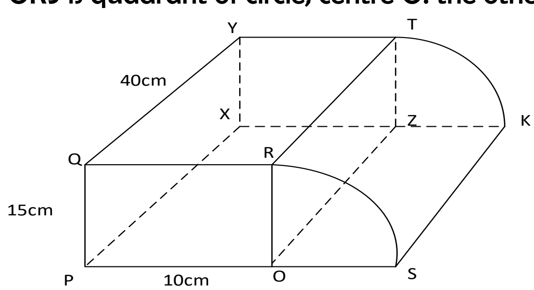
$= 1140cm^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 h = 1140$

$r = \sqrt[2]{\frac{1140x3}{20\pi}} = 7.38cm$

4. The diagram below shows a piece of wood of uniform cross section PQRS in which OPQR is a rectangle and ORS is quadrant of circle, centre O. the other rectangles are PQYX and PXZS




Given that PQ$= 15cm$, PO$= 10cm$ and QY$= 40cm$. calculate
   (i) Area of cross section PQRS
   (ii) Volume of the wood
   (iii) Total surface area of the piece of wood
   Take $\pi = 3.14$

**Solution**

(i) Area of cross section$= 10x15 + \frac{1}{4}(3.14x15^2)$

$= 150 + 176.625$

$= 326.625cm^2$

(ii) Volume $= area\ of\ cross - section x\ l$

$= 326.625x40 = 13065cm^3$

S.A of 5 rect. $= 2(10x15) + 2(40x10) + 40x15$

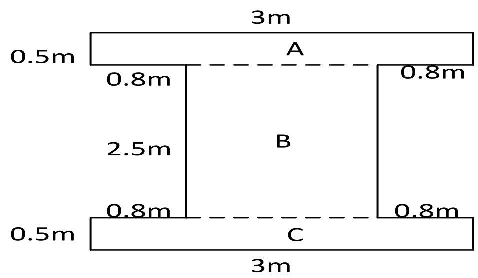
$= 1700cm^2$

S.A of curved surface of $\frac{1}{4}$ cylinder$= \frac{1}{4}(2\pi rh)$

$= \frac{1}{4}(2x3.14x15x40) = 942cm^2$

T.S.A of the wood= $2x176.625 + 1700 + 942$ $= 2995.25cm^2$

5. The diagram below shows the cross-section of a structure used as part of a building plan and the structure is 6m long.




Calculate;

(i) The cross-sectional area of the structure
(ii) The volume of the material used to fill the structure

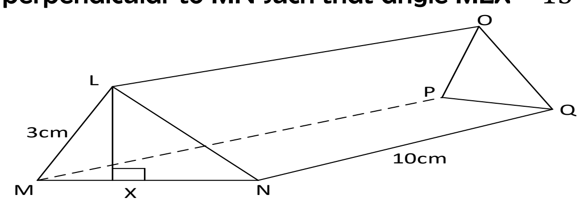
**Solution**

Area of A= $3x0.5 = 1.5m^2$
Area of B= $1.4x2.5 = 3.5m^2$
Area of C= $3x0.5 = 1.5m^2$

Area of cross-section= $1.5 + 3.5 + 1.5 = 61.5m^2$
Volume of structure= $6.5x6 = 39m^3$

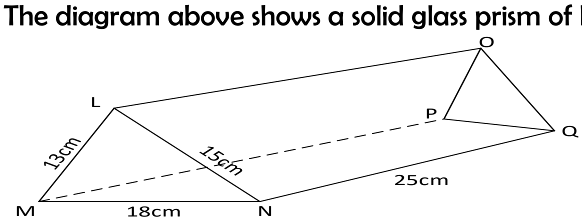
**Exercise**

1. Find the volume used to make a long cylindrical pipe 6m long whose external and internal radii are 6.16m and 7cm respectively. **An(** $= 7260cm^3,)$
2. The diagram above shows a triangular prism LMNOPQ of length 10cm and edge LM= $3cm$. LX is perpendicular to MN such that angle MLX= $15°$ and angle MLN= $85°$. Find the volume of the prism




**An(** $= 126.60cm^3,)$

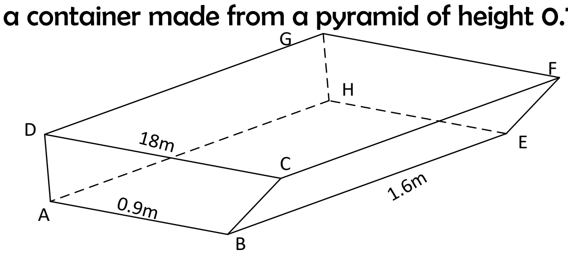
3. The diagram above shows a solid glass prism of length 25cm.




Find the;
(i) Area of cross-section
(ii) volume of the prism

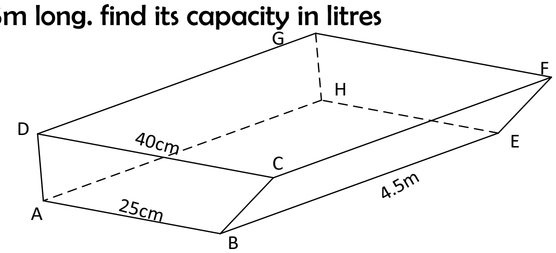
**An(** $(i) = 95.92cm^2 (ii) = 2398cm^3,)$

4. The figure below represents a container made from a pyramid of height 0.7m. find its capacity



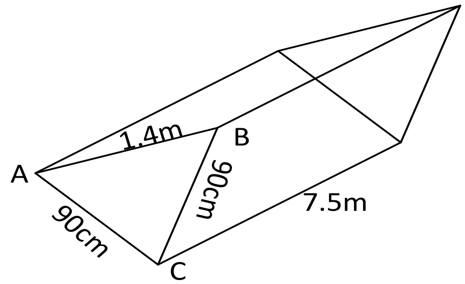

**An(** 1.176$m^3$)

5. The figure below represents a trough which is 40cm wide at the top and 25cm wide at the bottom. The trough is 20cm deep and 4.5m long. find its capacity in litres




**An(** 292.5 litres)

6. The diagram below shows a trough which has a triangular cross-section and is 7.5m long. Triangle ABC is an isosceles triangle in which $AB = 1.4m$ and $AC = BC = 90cm$

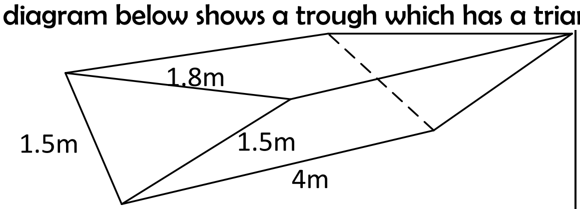



(a) Calculate the;
   (i) Cross-sectional area of the trough
   (ii) capacity of the trough in litres

(b) The trough is used to water cows in a farm. Each cow drinks 2.5 litres of water at a time for 3 times a day. If the trough is initially full, find the number of days it would take a herd of 44 cows to drink all the water in the trough.

**An**( $(i) = 3960cm^2, (ii) = 2970\ litre, (b) = 9\ days$)

7. The diagram below shows a trough which has a triangular cross-section and is 4m long.



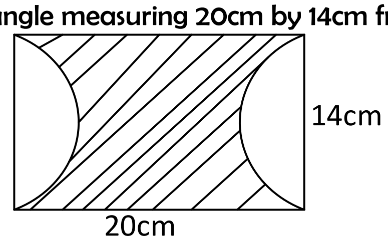

(a) Calculate the;
   (i) Cross-sectional area of the trough
   (ii) capacity of the trough in litres

(b) The trough is initially one-third full of water and is filled by a pipe which delivers water at a rate of 0.8 litres per second. How long does it take to fill the trough?

(c) The trough in (a) above is initially half full of water. Water is added to the trough using a cylindrical bucket of radius 15cm and height 30cm. how many bucketful of water must be added in order to fill the trough

**An**( $(i) = 1.08m^2, (ii) = 4320\ litre, (b) = 1\ h, (c) = 102$)

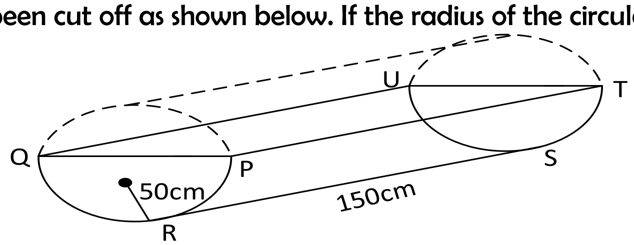
8. The diagram below shows a steel girder 5m long used in construction work. The cross-section consists of a rectangle measuring 20cm by 14cm from which two similar semi-circles have been removed




Calculate;
   (i) The cross sectional area of the girder
   (ii) Volume of the girder

**An**( $(i) = 154cm^2, (ii) = 63000cm^3$)

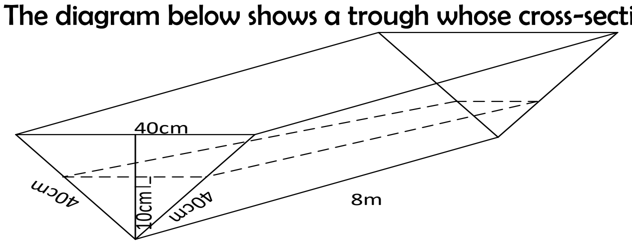
**9.** The diagram below shows a hollow right cylinder PQRSTU of negligible thickness. Part of which has been cut off as shown below. If the radius of the circular end is 50cm, RS= $150cm$ and PQ=TU= $20cm$




Find $(i)$ the area of the cross-section PQR
   $(\boldsymbol{ii})$ How much water in litres would fill this container? Use $(\pi = 3.14)$

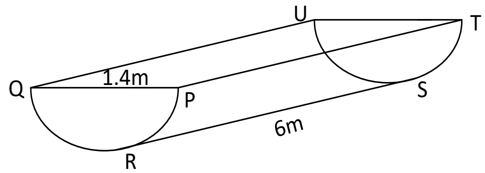
**An**( $(i) = 7841.44cm^2, (ii) = 1176.22cm^3$)

10. The diagram below shows a trough whose cross-section is an equilateral triangle of side 40cm.




The trough which is 8m long initially contains water to a depth of 10cm. water flows into the trough at the rate of 20 litres per minute. Calculate the time in minutes it takes to fill the trough. **An**(25min)

**11.** The diagram below shows a trough which is 6m long and whose cros-section is a semicircle of diameter 1.4m




(a) Find

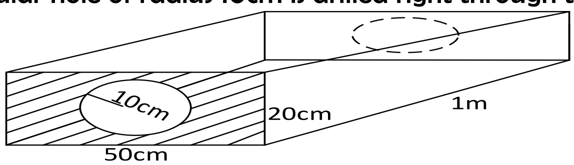
$(i)$ the area of the cross-section of the trough

$(ii)$ capacity of the trough in litres

(b) The trough is nitialy empty and is filled by a pump which pumps water at the rate of 1.54 liters per second. Find the time in minutes it takes to fill the trough

**An**( $(i) = 7700cm^2, (ii) = 4620\ litres, (b) = 50\ min)$

12. The diagram below shows a wooden structure measuring 1m long, 50cm wide and 20cm high. A circular hole of radius 10cm is drilled right through the as shown below. Take $\pi = 3$.



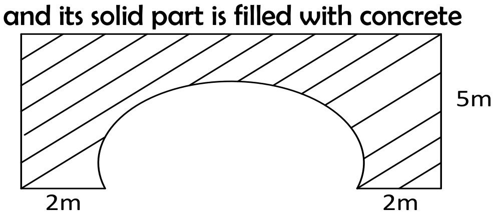

Calculate

(i) The cross-sectional area of the wood in the block

(ii) Volume of the wood

**An**( $(i) = 685.8cm^2, (ii) = 68580cm^3)$

13. The diagram below shows the cross-section of a bridge with a solid part and a tunnel through which the river flows. The tunnel is 8m long and its cross-section is a semi- circle of radius 3.5m. The bridge is 5m high and its solid part is filled with concrete



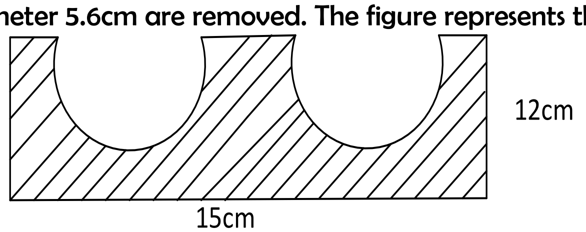

Calculate

(i) The cross-sectional area of the solid part

(ii) Volume of the concrete used to fill the solid part

**An**( $(i) = 35.75m^2, (ii) = 286m^3)$

14. The diagram below shows a rectangle measuring 15cm by 12cm form which two semicircles each of a diameter 5.6cm are removed. The figure represents the cross section of a wooden block 10.5m long




Calculate

(i) The cross-sectional area of the block

(ii) Volume of the wood in the block

**An**( $(i) = 155.4m^2, (ii) = 0.163cm^3)$

15. A solid metal sphere of diameter 30cm is melted down and recast into a hollow metal pipe of internal radius 3.5cm and external radius of 4cm. In the process 10% of the metal is lost. Calculate;
    (a) The volume of the metal in the sphere
    (b) Volume of metal used to make the pipe
    (c) Cross-sectional area of the metal in the pipe
    (d) Length of the pipe (take $\pi = 3.14$)

    **An**( $(i) = 14130cm^2, (ii) = 12717cm^3, (iii) = 11.775cm^2, (iv) = 10.8m)$

16. A solid metal sphere of diameter 15cm is melted down and recast into a hollow metal cylindrical pipe of internal radius 2.8cm and external radius of 3cm. In the process 12% of the metal is lost. Calculate;
    (a) Volume of metal used to make the pipe
    (b) Cross-sectioanl area of the metal in the pipe
    (c) Length of the pipe (take $\pi = 3.14$)

    **An**( $(i) = 1554.3cm^3, (ii) = 3.6424cm^2, (iii) = 4.27m)$

17. An open cylindrical reservoir of internal diameter 40m and depth 5m is constructed using reinforced concrete for walls and foundation. The walls are 2m thick while the foundation is laid to a depth of 2.5m.
    (a) Calculate;

(i) volume of the concrete used to construct the reservoir

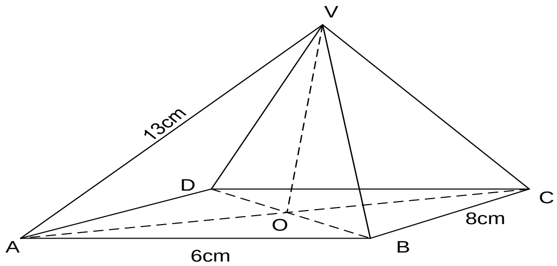
(ii) The capacity of the reservoir

(b) Starting with the full reservoir, water is consumed at the rate of $45800$ litres per day. Assuming no more water is added, calculate the number of days it takes to use up all the water in the reservoir

**An(** $(i) = 4105m^3, (ii) = 6284m^3, (iii) = 137\ days)$

**THREE DIMENSIONAL GEOMETRY(3-D)**

1. The figure below shows a right pyramid with the vertex V and edges VA, VB, VC, VD each 13cm long. The base ABCD is a rectangle of length 8cm and width 6cm




Calculate

(i) Length AC
(ii) height OV of the pyramid
(iii) Angle between VB and the base
(iv) Angle between the planes VAB and ABCD
(v) Angle between the planes VBC and the base
(vi) Volume of the pyramid

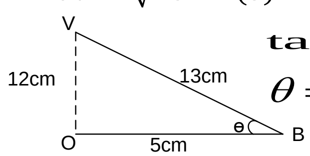
**Solution**

(i) $AC = \sqrt{8^2 + 6^2} = \sqrt{100} = 100cm$

(ii) $OC = \frac{10}{2} = 5cm$

$OV = \sqrt{13^2 - (5)^2} = \sqrt{144} = 12cm$

(iii)

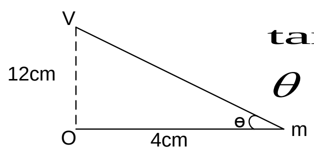



$\tan\theta = \frac{12}{5}$

$\theta = \tan^{-1}(2.4)$

$\theta = 67.38^0$

(iv)

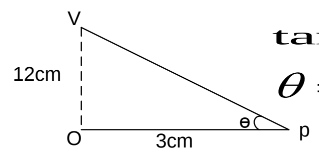



$\tan\theta = \frac{12}{4}$

$\theta = \tan^{-1}(3)$

$\theta = 71.57^0$

(v)

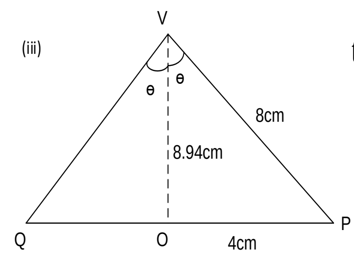



$\tan\theta = \frac{12}{3}$

$\theta = \tan^{-1}(4)$

$\theta = 75.96^0$

(iii)




$\tan\theta = \frac{4}{8.94}$

$\theta = \tan^{-1}(0.447)$

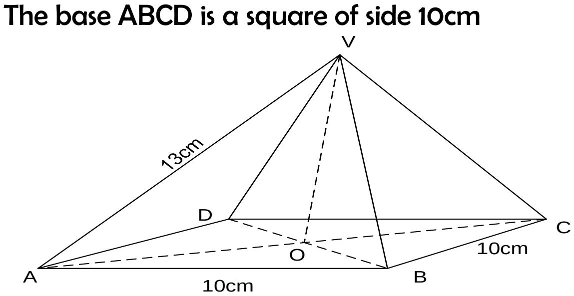
$\theta = 24.11^0$

$Angle = 2x24.11^0 = 48.22^0$

(iv) $V = \frac{1}{3}x\ base\ areaxh = \frac{1}{3}x8x6x12$

$= 192cm^3$

2. The figure below shows a right pyramid with the vertex V and edges VA, VB, VC, VD each 13cm long. The base ABCD is a square of side 10cm




Calculate

(i) Length AC
(ii) height of the pyramid
(iii) Cute angle between VB and the base ABCD
(iv) Cute angle between the planes BVA and ABCD
(v) Volume of the pyramid

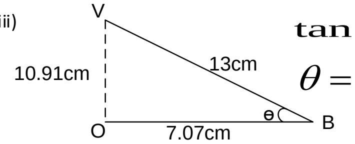
**Solution**

(i) $AC = \sqrt{10^2 + 10} = \sqrt{200} = 14.14cm$

(ii) $OC = \frac{14.14}{2} = 7.07cm$

$OV = \sqrt{13^2 - (7.07)^2} = 10.91cm$

(iii)

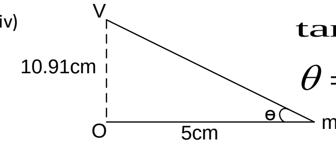



$\tan\theta = \frac{10.91}{7.07}$

$\theta = \tan^{-1}(1.543)$

$\theta = 57.06^0$

(iv)




$\tan\theta = \frac{10.91}{5}$

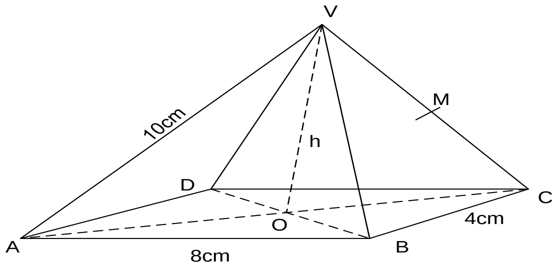
$\theta = \tan^{-1}(2.182)$

$\theta = 65.38^0$

(iv) $V = \frac{1}{3}x\ base\ areaxh = \frac{1}{3}x10x10x10.91$

$= 363.67cm^3$

3. The figure below shows a right pyramid with the vertex V and edges VA, VB, VC, VD each 10cm long. The base ABCD is a rectangle of length 8cm and width 4cm and M is the midpoint of CV




Calculate
(i) Vertical height h of the pyramid
(ii) Angle between the planes VBC and the base ABCD
(iii) Angle between the planes VBC and VAD
(iv) Volume of the pyramid

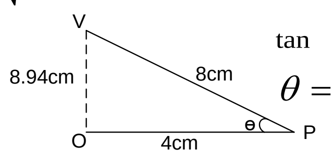
**Solution**

(iii) $AC = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5}cm$

$$OC = \frac{4\sqrt{5}}{2} = 2\sqrt{5}cm$$

$$h = \sqrt{10^2 - \left(2\sqrt{5}\right)^2} = \sqrt{80} = 8.94cm$$

(ii)

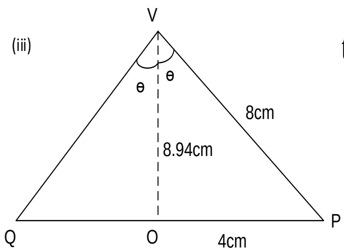



$$\tan \theta = \frac{8.94}{4}$$

$$\theta = \tan^{-1}(2.235)$$

$$\theta = 65.9^0$$

(iii)




$$\tan \theta = \frac{4}{8.94}$$

$$\theta = \tan^{-1}(0.447)$$

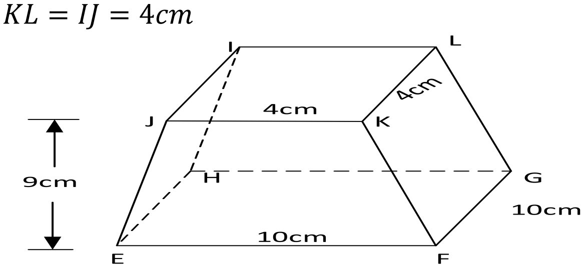
$$\theta = 24.11^0$$

$$Angle = 2x24.11^0 = 48.22^0$$

(iv) $V = \frac{1}{3}x\ base\ areaxh = \frac{1}{3}x8x4x8.94$

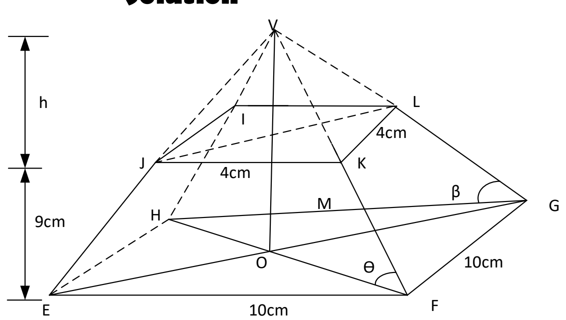
$$= 95.467cm^3$$

4. The diagram below EFGHIJKL is a square base frustrum whose dimensions are shown. The perpendicular ehigth of the frustrum is 9cm. given that $EF = FG = GH = HE = 10cm$ and $JK = KL = IJ = 4cm$




calculate;
(a) Vertical height of the original pyramid
(b) Angle between the line FK and the base EFGH
(c) Angle between the line LG and EF
(d) Volume of the frustrum

**Solution**




(i) $l.s.f = 4:10$

$$h:(9+h)$$

$$36 + 4h = 10h$$

$$h = 6cm$$

height $= 6 + 9 = 15cm$

(ii) $\Delta OVF: HF = \sqrt{10^2 + 10^2} = 10\sqrt{2}cm$

$$OF = \frac{1}{2}xHF = \frac{1}{2}x10\sqrt{2} = 5\sqrt{2}cm$$

$$tan\theta = \frac{15}{5\sqrt{2}}$$

$$\theta = 64.76°$$

(iii) $\Delta OVM: VM = \sqrt{15^2 + 5^2} = 5\sqrt{10}cm$

$$tan\beta = \frac{5\sqrt{10}}{5}$$

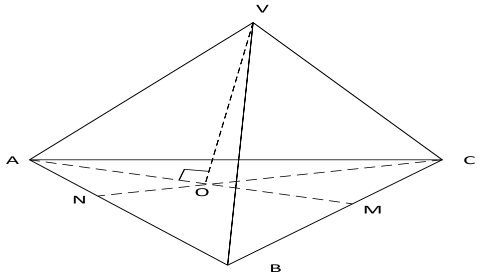
$$\beta = 72.45°$$

(iv) Volume of frustrum$= V_{big\ pyramid} - V_{small\ pyramid}$

$$= \frac{1}{3}x10x10x15 - \frac{1}{3}x4x4x6$$

$$= 468cm^3$$

5. The figure below shows a tetrahedron. The length of each edge is 8cm. O is the centre of triangle ABC




Calculate;

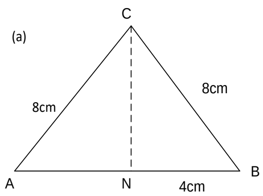
(a) The length VO

(b) The angle between the line AV and the plane ABC

(c) The angle between the planes ABC and VBC

(d) Volume of the tetrahedron

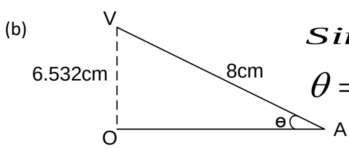
**Solution**




$$NC=\sqrt{8^2-4^2}$$
$$NC=\sqrt{48}$$
$$NC=6.928cm$$

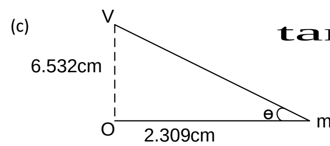
$$OC=\frac{2}{3}NC=\frac{2}{3}x6.928=4.619cm$$

$$\Delta OVC: VO=\sqrt{8^2-4.619^2}=6.532cm$$




$$Sin\theta=\frac{6.532}{8}$$
$$\theta=\sin^{-1}(0.8165)$$
$$\theta=54.74^0$$

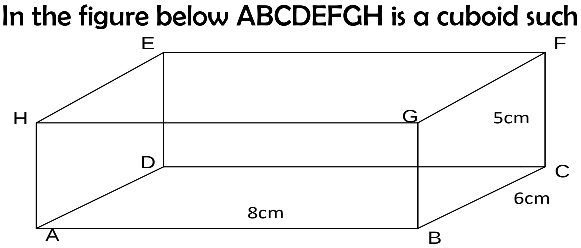
$$OM=\frac{2}{3}AM=\frac{1}{3}x6.928=2.309cm$$




$$\tan\theta=\frac{6.532}{2.309}$$
$$\theta=70.53^0$$

$$V=\frac{1}{3}x\ base\ areaxh=\frac{1}{3}x\left(\frac{1}{2}x6.928x8\right)$$
$$=9.237cm^3$$

6. In the figure below ABCDEFGH is a cuboid such that $AB=8cm$, $BC=6cm$ and $CF=5cm$




Calculate

(a) Length

(i) AC

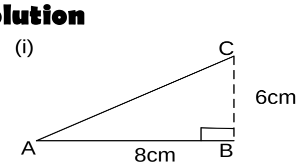
(ii) AF

(b) Angle AF makes with the plane ABCD

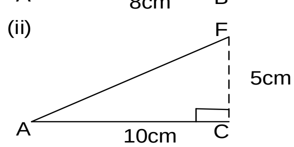
(c) Angle AEFB makes with the base ABCD

(d) Volume of cuboid

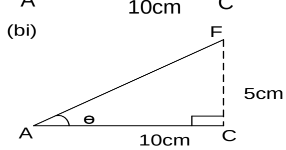
**Solution**



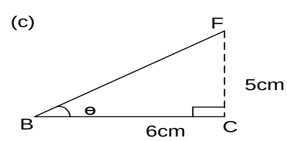

$$AC=\sqrt{8^2+6^2}$$
$$AC=\sqrt{100}$$
$$AC=10cm$$




$$AF=\sqrt{10^2+5^2}$$
$$AC=\sqrt{125}$$
$$AC=11.18cm$$




$$\tan\theta=\frac{5}{10}$$
$$\theta=\tan^{-1}(0.5)$$
$$\theta=26.57^0$$



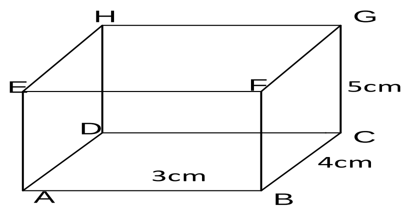

$$\tan\theta=\frac{5}{6}$$
$$\theta=\tan^{-1}(0.833)$$
$$\theta=39.8^0$$

$(d)$ Volume= $lwh$

Volume= $8x5x6$

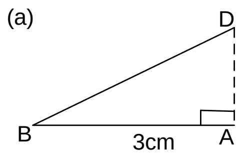
$$volume=240cm^2$$

7. In the figure below ABCDEFGH is a cuboid such that $AB=8cm$, $BC=6cm$ and $CF=5cm$

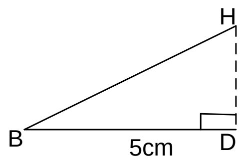



Calculate
(i) Length HB
(ii) Angle AH makes with the plane ABCD
(iii) Angle FDB
(iv) Angle between AFC and ABCD
**An**(63.4°)

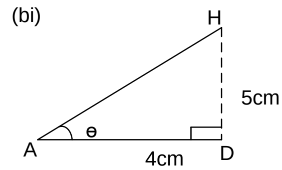
**Solution**



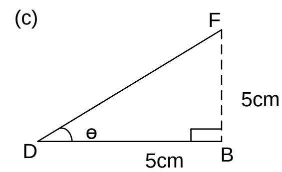

$$DB = \sqrt{4^2 + 3^2}$$
$$DB = \sqrt{25}$$
$$DB = 5cm$$




$$DB = \sqrt{5^2 + 5^2}$$
$$DB = \sqrt{50}$$
$$DB = 7.07cm$$

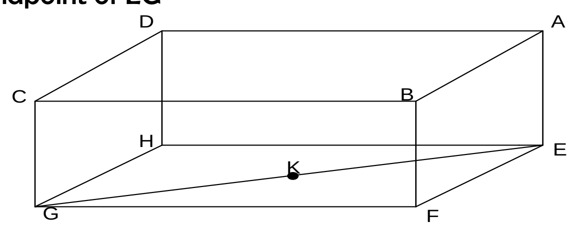



$$\tan\theta = \frac{5}{4}$$
$$\theta = \tan^{-1}1.25$$
$$\theta = 51.34^0$$



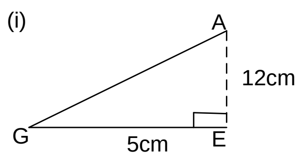

$$\tan\theta = \frac{5}{5}$$
$$\theta = \tan^{-1}(1)$$
$$\theta = 45^0$$

8. In the figure below ABCDEFGH is a cuboid with a square base. $EG = 5cm$, $AE = 12cm$ and $K$ is the midpoint of EG

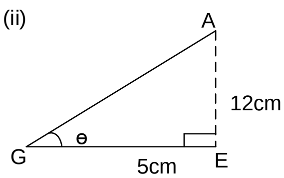



Calculate
(v) Length AG
(vi) Size of angle between AG and base
(vii) Area of triangle $AGK$
(viii) Length CF
(ix) Volume of cuboid

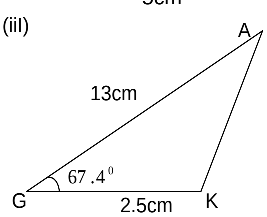
**Solution**



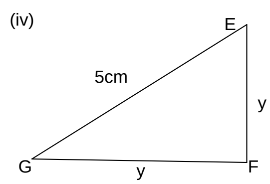

$$AG = \sqrt{5^2 + 12^2}$$
$$AG = \sqrt{169}$$
$$AG = 13cm$$



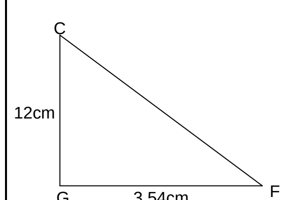

$$\tan\theta = \frac{12}{5}$$
$$\theta = \tan^{-1}(2.4)$$
$$\theta = 67.4^0$$




$$Area = \frac{1}{2}x13x2.5\sin 67.4$$
$$Area = 15cm^2$$




$$5^2 = y^2 + y^2$$
$$25 = 2y^2$$
$$12.5 = y^2$$
$$y = \sqrt{12.5} = 3.54cm$$

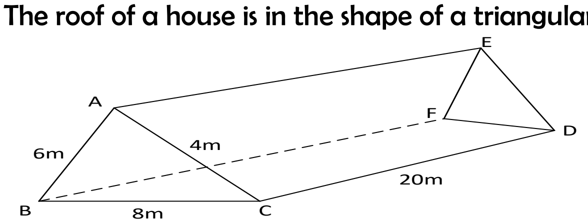



$$CF = \sqrt{3.54^2 + 12^2}$$
$$CF = \sqrt{156.5316}$$
$$y = 12.51\ cm$$

(v) Volume= $lwh$
Volume= $3.54x3.54x12$
$volume = 150.38cm^2$

**EXERCISE**

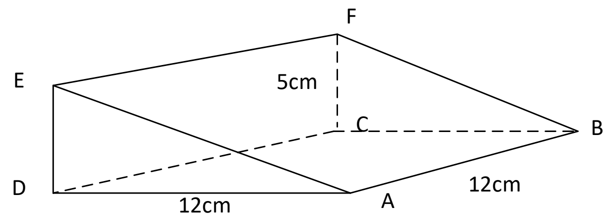
1. The roof of a house is in the shape of a triangular prism, as shown below




(a) The angle between faces FBAE and CDFB
(b) Volume of the spaces occupied by the roof
(c) The angle between plane DAF and CDFB

**An**$((i) = 28.96°, (ii) = 232.38m^3, (iii) = 8.25°$

2. The diagram below shows a wedge in which $AB = BC = 12cm$ and $CF = 5cm$. The wedge is painted on all surfaces

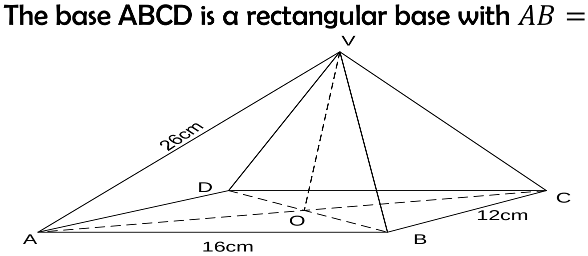



Determine;

(a) Total surface area of the wedge painted
(b) Angle made by the slanting plane ABFE and the horizontal plane ABCD
(c) The angle between the slanting plane ABFE and the vertical plane CDEF

**An**$((a) = 420cm^2, (b) = 22.6°, (c) = 67.4°$

3. The base of a right pyramid is a rectangle 10.0cm by 8.0cm and the slant edges are each 13cm. Calculate the;
   (a) Total surface area of the pyramid
   (b) Angle between two opposite slanting side of the pyramid whose base length is 10.0cm
   **An**$((i) = 289.96cm^2, (ii) = 39°,$
4. A rectangular swimming pool is constructed such that when the pool is completely full, the shallow end is 1m deep and the deeper end is 4m deep. The pool is 25m long from the shallow end to the deep end and 20m wide
   (a) Calculate the;
   (i) Inclination of the floor of the swimming pool to the horizontal
   (ii) volume of the water in $m^3$ that can fill the pool
   (b) starting with the pool empty, a tap which delivers water at a rate of 400 litres per minute is used to fill the pool. How long in hours will the pool take to fill
   **An**$((a)(i) = 6.84°, (ii) = 1250m^3, (iii) = 52h$
5. The figure below shows a right pyramid with the vertex V and edges VA, VB, VC, VD each 26cm long. The base ABCD is a rectangular base with $AB = 16cm, BC = 12cm$

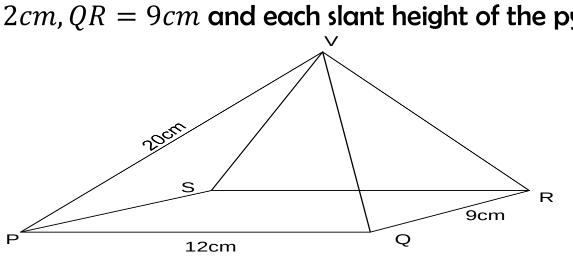



Calculate

(i) height of the pyramid
(ii) angle between VB and the base ABCD
(iii) Cute angle between the planes BCV and ABCD

**An**$((i) = 24\ cm, (ii) = 67.4°, (iii) = 71.6°$

6. The figure below shows a right pyramid VPQRS which stands on a rectangular base PQRS side $PQ = 12cm, QR = 9cm$ and each slant height of the pyramid is 20cm long

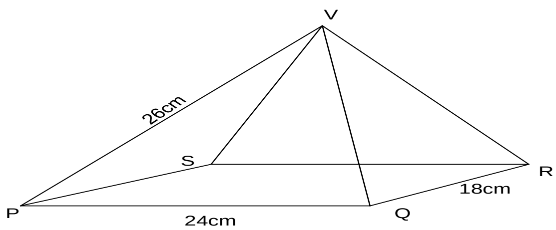



Calculate

(i) Length PR
(ii) Vertical height of the pyramid
(iii) Volume of the pyramid
(iv) angle between plane VPQ and the PQRS

**An**$((i) = 15\ cm, (ii) = 18.54cm, (iii) = 667.4cm^3\ (iv) = 76.36°$

7. The figure below shows a right pyramid stands on a rectangular base ABCD. V is the vertex of the pyramid and $VA = VB = VC = VD = 26cm$ . Side $AB = 24cm, BC = 18cm$

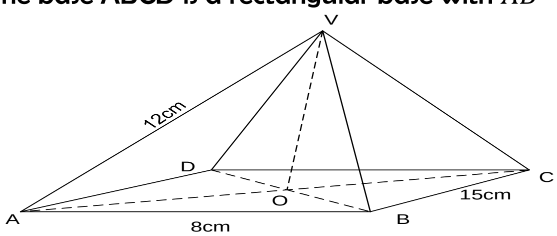



Calculate

(i) Length AC
(ii) Vertical height of the pyramid
(iii) angle between plane VA and the ABCD
(iv) angle between plane VBC and the ABCD

**An**$((i) = 30cm, (ii) = 21.24m, (iii) = 54.77°,\ (iv) = 60.54°$

8. The figure below shows a right pyramid with the vertex V and edges VA, VB, VC, VD each 12cm long. The base ABCD is a rectangular base with $AB = 8cm, BC = 15cm$

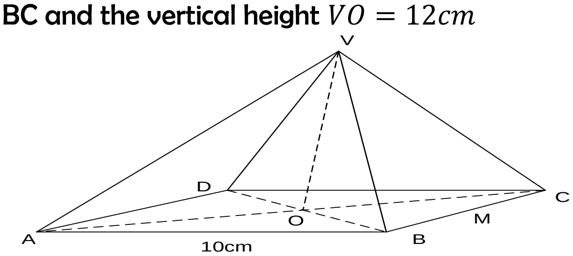



Calculate

(i) Vertical height of the pyramid
(ii) Volume of the pyramid
(iii) angle between plane VBC and the base ABCD
(iv) Cute angle between the line VA and ABCD

**An**$((i) = 8.5\ cm, (ii) = 338.8cm^3,\ (iii) = 64.8°, (iv) = 45.1°$

9. The figure below shows a right pyramid standing on a square base of side 10cm. M is the mid-point of BC and the vertical height $VO = 12cm$

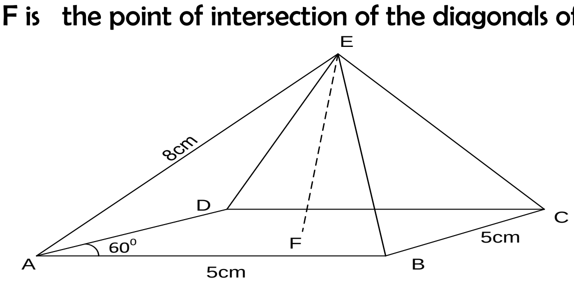



Calculate

(i) The length of the projection VA on the plane ABCD
(ii) Size of the angle between VA and the plane ABCD
(iii) angle between plane VBC and the base ABCD
(iv) Total surface area of pyramid

**An**$((i) = 7.07cm,\ (ii) = 59.49°, (iii) = 67.38°, (iv) = 360cm^2$

10. The figure below shows a right pyramid with the vertex V and edges VA, VB, VC, VD each 26cm long. The base ABCD is a rhombus of side 5cm and whose acute angle is $60°$. $AE = DE = CE = BE = 8cm.$ F is the point of intersection of the diagonals of the rhombus. Find

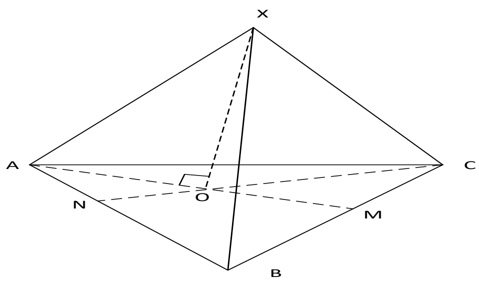



Calculate

(i) Length EF
(ii) angle AEB
(iii) angle between each of the slanting planes and ABCD

**An**$((i) = 718cm, (ii) = 36.42°,\ (iii) = 70.8°$

9. The figure below shows a right pyramid standing XABC standing on a triangular base ABC. Each of the triangular faces of the pyramid is an equilateral triangle of side 12cm. M is the midpoint of BC and the vertical height OX of the pyramid is 9.6cm

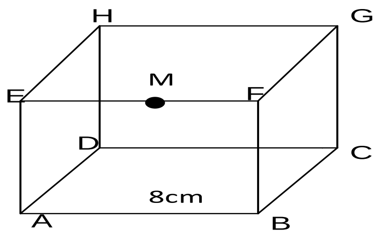



Calculate;

(a) Volume of the pyramid
(b) Total surface area of the pyramid
(c) The angle between the planes XBC and ABC

**An**$((i) = 199.5cm^3, (ii) = 249.4cm^2,$
$(iii) = 67.4°$

11. The diagram below shows a cube ABCDEFGH of sides 8cm and $EM = MF$. A tetrahedron AMHE is cut off the cube.

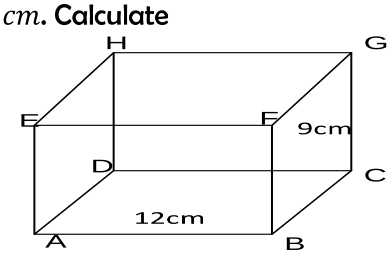



Find the;

(i) Area of triangle HAM
(ii) Angle between plane HAM and the plane AEHD
(iii) Volume of the remaining part of the cube after the tetrahedron has been cut off

**An**$((i) = 39.23cm^2, (ii) = 35.3°, (iii) = 384cm^3$

12. The diagram below shows ABCDEFGH is a rectangular box with square ends, BCFG and ADEH of side 9cm, AB= $12cm$. Calculate

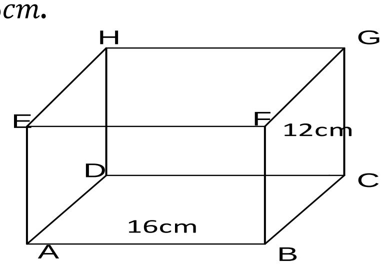



(i) Length AC and AG
(ii) Angle between the line AG and the plane ABCD
(iii) Angle between the planes DEG and ADEH

**An**$((i)AG = 17.5cm, AC = 15cm, (ii) =$
$31°, (iii) = 43.31°$

Find the;

13. The diagram below shows ABCDEFGH is a rectangular box with square ends, BCFG and ADEH of side 12cm, AB= $16cm$.

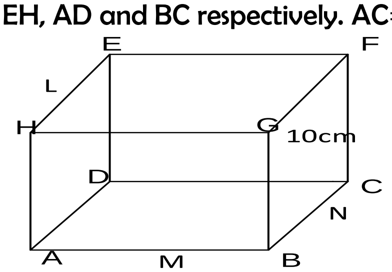



Find the;

(i) Length BD and BH
(ii) Angle between the line BH and the plane ABCD
(iii) Angle between the planes CFH and BCGF

**An**$((i)BH = 23.3cm, BD = 20cm$
$(ii) = 31°, (iii) = 62.1°$

14. The diagram below shows a cuboid ABCDEFGH with a square base. The point L, M and N are the mid-points of EH, AD and BC respectively. AC= $7.5cm$, and $CF = 10cm$.

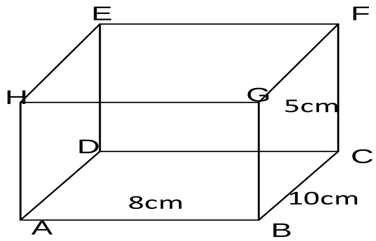



Find the;

(i) Length BC and LN
(ii) Angle between the plane EHBC and plane ABCD
(iii) Angle between the line AF and ABCD

**An**$((i)BC = 5.3cm, LN = 11.3cm$
$(ii) = 62.2°, (iii) = 53.1°$

15. The diagram below shows ABCDEFGH shows an open cuboid in which AB= $8cm$, BC= $10cm$ and CF= $5cm$




Find the;

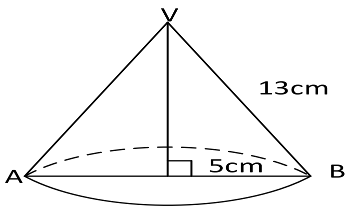
(i) Angle between the line BE and the plane ABCD

(ii) Angle between the planes ABEF and ABCD

(iii) Angle between the line AE and the plane EFCD

**An**$((i) = 21.32°,\ \ (ii) = 26.57°, (iii) = 63.43°$

10. The figure below shows a right circular cone AVB. The radius of the base is 5cm and the slanting edge 13cm



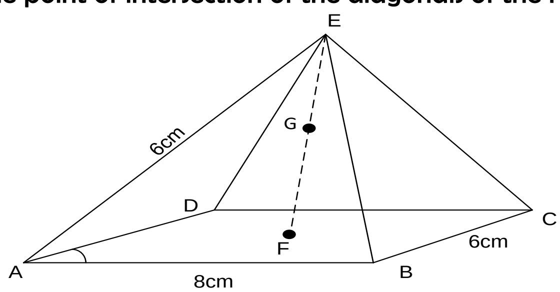

(a) Calculate angle AVB

(b) Find the
 (i) Volume of the cone
 (ii) Total surface area of the closed cone

$\pi = 3.142$

**An**$((a) = 45.24°, (b)(i) = 314.3m^3, (iii) = 282.78cm^2$

11. The figure below shows a right pyramid with the vertex V and edges VA, VB, VC, VD each 6cm long. The base ABCD is rectangular of side 8cm by 6cm. Slanting edges $AE = DE = CE = BE = 6cm$. F is the point of intersection of the diagonals of the rectangular. Find




Calculate

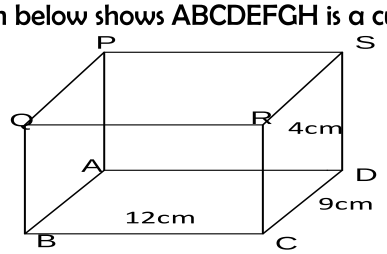
(i) Angle AEC

(ii) Length EF and AG

(iii) angle between each of the slanting planes and ABCD

**An**$((i) = 112.89°, (ii) = 3.32cm, 5.47cm\ (iii) = 39.69°$

12. The diagram below shows ABCDEFGH is a cuboid




Find the;

(i) Length QC and PC

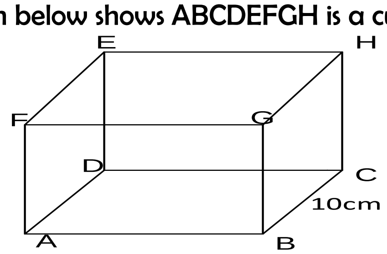
(ii) Angle PCQ

(iii) Angle between the planes PQC and PQRS

**An**$((i) = 12.65cm, 15.52cm\ (ii) = 35.43°,$

$(iii) = 18.4°$

13. The diagram below shows ABCDEFGH is a cub of side 10cm




Find the;

(i) Length BF and BE

(ii) Angle between BE and the face ABGF

(iii) Angle between the planes BDF and ABCD

**An**$((i) = 14.14cm, 17.32cm\ (ii) = 35.26°,$

$(iii) = 54.7°$